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PAMPHLET M-1

MATHEMATICS

ELEMENTARY ARITHMETIC

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PAMPHLET M-1

MATHEMATICS

ELEMENTARY ARITHMETIC

MATHEMATICS.

The first lesson in this paper consists of......	questions 1 to 8,	inclusive.
The second lesson..	questions 9 to 16	"
The third lesson..	questions 17 to 21	"
The fourth lesson..	questions 22 to 26	"
The fifth lesson...	questions 27 to 31	"

These should be answered one lesson at a time.

MATHEMATICS.

The following signs are frequently used in connection with the different computations and formulas, so that their meaning and applications should be made familiar:

$+$ Plus, indicates addition; thus, $8 + 5$ are 13.

$-$ Minus, indicates subtraction; thus, $8 - 5$ are 3.

$\times$ Times or multiplied by, indicates multiplication; thus, 8×5 are 40.

$\div$ Divided by, indicates division: thus, $20 \div 5$ are 4.

$=$ Equals or equal to, indicates equality; thus, $4 + 5 = 9$, or 3 ft. $= 1$ yd.

$\pm$ Plus or minus, indicates a positive or negative quantity.

$\mp$ Minus or plus, indicates a negative or positive quantity.

ab or $a \cdot b = a \times b$, indicates "a" multiplied by "b," and may be written in any of the three ways.

$\frac{a}{b} = a \,/\, b = a \div b$, indicates that the quantity "a" is to be divided by the quantity "b."

$.2 = \frac{2}{10}$ $.002 = \frac{2}{1000}$ Decimal fractions.

( ), [ ], { }, called parenthesis, bracket and brace, respectively, have the same meaning and indicate that the operations within them must be performed first. When two or more are used, the innermost must be effected first; thus, $8\ (6 - 3)$ shows that the subtraction must be performed first before multiplying by 8. When a minus sign, —, is just before the bracket and it is desired to remove the bracket to simplify the quantity, then the plus and minus signs inside this bracket would be changed as follows:

$$5\ [6 - (3 - \tfrac{1}{2})] \text{ would be written}$$
$$5\ [6 - 3 + \tfrac{1}{2}] \text{ or } 5 \times 3\tfrac{1}{2} \text{ equals } 17\tfrac{1}{2}.$$

———, called the vinculum, is used for same purpose as the parentheses, but most frequently with the radical sign $\sqrt{}$.

$\sqrt{}$, called the radical sign, and indicates that the square root of the quantity under it is to be found.

$\sqrt[3]{\ }$, $\sqrt[4]{\ }$, indicates that the cube root and the fourth root, respectively, are to be found.

a^2, a^3, is read "a" squared and "a" cubed, meaning that "a" is to be used twice as a multiple or three times, respectively. It is same as $a \times a$, or $a \times a \times a$. The figures 2 and 3 as used here are called exponents.

$a^{\frac{2}{3}} = \sqrt[3]{a^2}$, $a^{\frac{3}{2}} = \sqrt{a^3}$, fractional exponents, and are read "a" to the $\frac{2}{3}$ power and "a" to the $\frac{3}{2}$ power. The denominator of a fraction used as an exponent shows what root is to be found, and the numerator indicates the number of times the quantity is to be used as a multiple.

: means ratio or divided by; thus $2:4$ is read in the ratio of 2 to 4, or $\frac{2}{4} = \frac{1}{2}$.

: :: : proportion signs; thus, $2:4::8:16$, and is read 2 is to 4 as 8 is to 16.

Expressed as an equation, $\frac{2}{4} = \frac{8}{16}$.

$\therefore$ means therefore.

$>$ means greater than.

$<$ means less than.

$^\circ$ degrees of a thermometer or an arc.

$'$ minutes or feet.

$''$ seconds or inches.

$'$ $''$ $'''$ accents to distinguish letters or quantities; as, a' a'' a''', and read a prime, a second and a third.

a_1, a_2, a_3, subscripts to distinguish letters or quantities, and read a sub one, a sub two, etc.

a^{-2}, a^{-3}, negative exponents, indicates that the reciprocal of the quantity raised to the given power is to be taken. It is read "a" to the minus 2 power, "a" to the minus 3 power. These quantities are equal to $\frac{1}{a^2}$ and $\frac{1}{a^3}$.

π read pi, and is the ratio of the diameter to the circumference of a circle, and is equal to 3.1416, which is approximately $3\frac{1}{7}$.

It is assumed that the common operations, such as addition, subtraction, multiplication and division, in which whole numbers are involved are thoroughly familiar. In order to refresh the memory a short review of mixed numbers and fractions may not be out of place.

FRACTIONS.

In the fraction $\frac{3}{4}$ the numerator is 3 and the denominator is 4. The denominator shows the number of parts into which the thing is divided and the numerator the number of parts taken.

Common denominator is a denominator that will contain each of the denominators of several fractions without a remainder. The numerators of each fraction are then multiplied by the number of times its denominator is contained in the common denominator. For example, $\frac{1}{2}$, $\frac{3}{4}$, $\frac{7}{8}$ have a common denominator of 8, and $\frac{1}{2}=\frac{4}{8}$; $\frac{3}{4}=\frac{6}{8}$; $\frac{7}{8}=\frac{7}{8}$.

Reduce a fraction to its lowest terms. Divide both numerator and denominator by the greatest number that each will contain without a remainder; for example, in $\frac{12}{16}$ the greatest number that will thus divide both 12 and 16 is 4; so that $\dfrac{12 \div 4}{16 \div 4}=\frac{3}{4}$, which has the same value as the original fraction $\frac{12}{16}$.

Multiplying or dividing both numerator and denominator of any fraction by the same number does not change the value of the fraction.

A mixed number is one consisting of a whole number and a fraction; as, $4\frac{3}{8}$.

An improper fraction is one whose numerator is equal to or greater than its denominator; as, $\frac{35}{8}$. This may be reduced to a whole or mixed number by dividing the numerator by the denominator, the quotient being the whole number and the remainder being the numerator of the fraction having same denominator as the improper fraction; thus, $35 \div 8 = 4\frac{3}{8}$. A mixed number may be changed to an improper fraction by multiplying the whole number by the denominator of the fraction, adding the numerator and placing this sum over the denominator of the original fraction. For example, $3\frac{1}{2}=\dfrac{(2 \times 3)+1}{2}=\frac{7}{2}$.

Adding Fractions or Mixed Numbers: Fractions alone may be added by reducing them to a common denominator, as mentioned above, then adding the numerators and reducing the sum to a whole or mixed number. Mixed numbers are added by adding separately the whole numbers and the fractions and adding these sums; thus, $3\frac{1}{2}+4\frac{1}{4}=(3+4)+(\frac{1}{2}+\frac{1}{4})=7+(\frac{2}{4}+\frac{1}{4})=7\frac{3}{4}$.

Subtracting Two Fractions or Mixed Numbers: First reduce the fractions to common denominator and subtract the numerators; thus, $\frac{7}{8}''-\frac{1}{4}''=\frac{7}{8}-\frac{2}{8}=\frac{5}{8}''$. When mixed numbers are to be subtracted, subtract the whole numbers and fractions separately and place their remainders together; thus, $4\frac{1}{2}'' - 2\frac{1}{4}'' = (4'' - 2'') + (\frac{1}{2}'' - \frac{1}{4}'') = 2\frac{1}{4}''$, or if the minuend, or larger number, has the smaller fraction, as follows: $5\frac{1}{2}'' - 2\frac{3}{4}'' = (4'' + \frac{2}{2}'' + \frac{1}{2}'') - 2\frac{3}{4}'' = (4'' + \frac{4}{4}'' + \frac{2}{4}'') - 2\frac{3}{4}'' = 2\frac{3}{4}''$.

Multiply Fractions: The numerators should be multiplied together, also the denominators, and then divide the numerator by the denominator; thus,

$$\frac{3}{8} \times \frac{7}{12} \times \frac{1}{2} = \frac{3 \times 7 \times 1}{8 \times 12 \times 2} = \frac{21}{192} = \frac{7}{64}$$

When multiplying mixed numbers change them to improper fractions and proceed as above.

Divide Fractions: The divisor (the one by which you divide) is inverted, and then proceed as if to multiply; thus, $\frac{9}{15} \div \frac{3}{8} = \frac{9}{15} \times \frac{8}{3} = \frac{72}{45} = 1\frac{27}{45} = 1\frac{3}{5}$.

Decimal Fractions.

It is of great importance that a thorough knowledge of decimals be possessed, as they are constantly used in making calculations. They are much easier to use than common fractions, but in some cases the common fractions are most convenient.

The divisions of anything into tenths, hundredths, thousandths, etc. are called Decimal Divisions. One or more of these decimal divisions of a unit forms a Decimal Fraction, commonly called a Decimal. This term is derived from the Latin word "*decem*," meaning ten.

Since tenths are equal to ten times as many hundredths, hundredths equal to ten times as many thousandths, etc., decimals have the same laws of increase and decrease as whole numbers, the denominator being indicated by the position of the decimal point.

In this system of writing fractions, the value of a figure is decreased tenfold by each removal one place to the right from the decimal point. The first figure to the right of units or of the decimal point is read as tenths; thus 3.4 is read as three and four tenths. The second figure to the right of the decimal point or the one to the right of tenths is read as hundredths; thus 3.04 is read as three and four hundredths, or 3.24 is read three and twenty-four hundredths. The figure at the right of hundredths, or the third from the decimal point, is read as thousandths, etc.

A period placed before the decimal is called the Decimal Point.

Reading Decimals.

The whole number written before the decimal point is read same as usual; the word *and* always separates the whole number from the fraction when expressing the latter in terms of its denominator; the decimal is read as an integral number, and is expressed in terms of the denomination of the right-hand figure; thus 40.078 is read forty and seventy-eight thousandths.

The method of reading decimals often followed in business, and which is advisable to use in telephone conversation, is to repeat the figures comprising the whole number, call the decimal point merely "point," then repeat the figures following the decimal point. Then 40.078 would be read as four, oh, point, oh, seven, eight.

Adding Decimals: Always place the figures so that the decimal points come under each other. Add in usual way, placing the decimal point in the sum underneath the other points; thus,

$$\begin{array}{r} 48.756 \\ 301.28 \\ 10.3298 \\ \hline 360.3658 \end{array}$$

Annexing ciphers after a decimal fraction does not alter its value.

Each cipher inserted between the figures of a decimal and the decimal point is equivalent to dividing the fraction by ten.

The denominator of a decimal is read as if it were one, with as many ciphers annexed as there are figures after the decimal point.

Subtracting Decimals: Place the numbers with the decimal points underneath each other and subtract same as usual; thus,

$$\begin{array}{r} 482.56 \\ 39.375 \\ \hline 443.185 \end{array}$$

Add a cipher to first number, as that does not change its value and makes the operation clearer.

To reduce decimals having different denominators to similar decimals, annex ciphers after the figures so that all have the same number of decimal places, as this does not alter their value; thus .5, .25, .465 and 3.015 may be written as:

$$\begin{array}{rcr} .5 & = & .500 \\ .25 & = & .250 \\ .465 & = & .465 \\ 3.015 & = & 3.015 \end{array}$$

To change decimals to common fractions, write the number, omitting the decimal point, supply the proper denominator and reduce to its lowest terms; thus, $.075 = \frac{75}{1000} = \frac{3}{40}$.

To change common fractions to equivalent decimals, annex ciphers to the numerators and divide by the denominator. Point off as many places in the quotient as there are ciphers annexed.

Many times the division, changing a common fraction to a decimal, is not complete. In such cases it is customary to either write the remainder as a common fraction having for its numerator the remainder and its denominator the divisor; thus, $\frac{1}{3} = .333\frac{1}{3}$; or the sign $+$ may be used to show that the division is not complete and that it may be carried further; as, $.333+$.

Multiplication.

Two decimals are multiplied like whole numbers, and from the right of the product point off as many places as there are decimal places in both factors; thus, $4.417 \times .34 = 1.50178$.

$$\begin{array}{r} 4.417 \\ .34 \\ \hline 17668 \\ 13251 \\ \hline 1.50178 \end{array}$$

Division.

This is performed the same as if the decimals were whole numbers. If the dividend (the number to be divided) does not have as many figures as the divisor (the number to divide by), ciphers are annexed to the dividend until the division is carried as far as desired. Judgment should be used regarding how far it is advisable to carry out a decimal.

In the ordinary business computation four or five places are sufficient.

The quotient is pointed off by taking as many figures from the right for decimal as the number of decimal places in the dividend exceed those in the divisor. If the quotient does not contain a sufficient number of figures, ciphers should be prefixed to supply the deficiency; thus, $.0075 \div 1.25 = .006$.

$$\begin{array}{r} 1.25).00750(.006 \\ 750 \\ \hline \end{array}$$

Ratio and Proportion.

The relation of one number to another of the same kind is known as Ratio. This may be expressed in two ways, either by arithmetical or geometrical ratio; thus the ratio of 4 to 8 is expressed as $\frac{4}{8}$ or $\frac{1}{2}$, which is the geometrical ratio, and is the ratio always referred to unless otherwise mentioned. The constant difference between numbers in a series, as 4, 8, 12, 16, 20, 24, etc., which is 4, is the arithmetical ratio, but this is seldom used.

If the relation of one number to another is sought, the first number mentioned is the dividend, the second is the divisor; but if the relation between the two numbers is desired, either may be considered as the dividend or divisor.

The numbers so compared are known as the terms of the Ratio. The first number is the antecedent and the second the consequent.

Inverse Ratio.

When expressing the ratio between two quantities it is understood to mean the "Direct Ratio" or comparison between them in the order in which they are mentioned, unless otherwise specified. An Inverse or Reciprocal Ratio of two quantities changes the relative position of the terms of the ratio. In expressing as a fraction the ratio of 3 to 10, it would be written $\frac{3}{10}$, but the inverse ratio of 3 to 10 would be $\frac{10}{3}$.

Examples showing the application of "Inverse Ratio" are numerous. The resistance of electrical conductors of the same material varies inversely as their areas of cross-section. That is, if a conductor has twice the cross-sectional area, its resistance would be half that of the other of same material. The intensity of illumination on a surface varies inversely as the square of its distance from the source of the illumination; that is, if a paper is held a distance of 2 feet from a lamp, the illumination on the paper will be four times as intense as when it is held at the same angle and 4 feet away from the lamp, or in the inverse ratio of 2^2 to 4^2, which equals $\frac{16}{4}$ equals 4.

An equality of ratios is known as a Proportion; thus $1:2::6:12$ is a proportion, and is read one is to two as six is to twelve.

There must be four terms in a proportion, viz., two antecedents and two consequents. In the above proportion 1 and 6 are the antecedents and 2 and 12 are the consequents.

The first and last terms are also called the extremes, and the second and third the means. The product of the extremes is equal to the product of the means. From this it follows that the product of the means divided by either extreme will give the other extreme, and the product of the extremes divided by either mean will give the other mean.

By considering cause and effect as applied to proportion, a great many problems can be easily solved. Let the first term be considered as a cause and the second the effect produced by this cause, the third a similar cause and the fourth the effect produced by this second cause. This may be applied in the following problem: How high is a building that makes a shadow 162 feet long when a flag-staff whose height is known to be 60 feet casts a shadow 72 feet?

Solution: The flag-staff is the cause of the shadow and the shadow is the effect; similarly with the building and its shadow. Then we may state this as a proportion:

60 : 72 :: X : 162. The quantity X is the unknown quantity which we are to solve for. From the product of means and extremes we can easily find this:

$$\frac{60 \times 162}{72} = 135, \text{ the height of the building.}$$

In like manner the height of a stack or other object may easily be found.

Proportion may be written as an algebraic equation; as $a : b :: c : d$, which is an application of the same principle, namely, an equality of two ratios.

Many times the expression of a formula or rule may be more clearly seen and understood by stating it in the form of an equation; but some people have a peculiar idea that an equation, or anything of a similar nature pertaining to mathematics other than the simple processes, can be understood and applied only by those having an excellent understanding of algebra and higher mathematics. Such an idea is erroneous. Most formulas with which we shall have any occasion to deal can be solved by arithmetic.

The letters a, b, c and d mentioned in the proportion above are used to represent quantities, and may have any value whatever so long as the ratio of a to b remains the same as the quantities represented by the ratio c to d; this is also stated:

$\frac{a}{b} = \frac{c}{d}$, $ad = bc$, $a = \frac{bc}{d}$, $d = \frac{bc}{a}$, $b = \frac{ad}{c}$, $c = \frac{ad}{b}$, so that when any three of the quantities are known the missing number may be solved for.

Involution.

This is the process of multiplying a number by itself a certain number of times. The product thus obtained is said to be a certain power of the original number. If the number is multiplied by itself once, the result is called the square of the number; thus $3^2 = 9$ is read three squared equals nine. Nine is, therefore, the square of three.

The figure placed above and to the right of a number is called the *exponent* or *index* of the number, and indicates how many times the number will be taken as a factor.

When the square of a number is multiplied by the number, the result is the cube of the number. The expression of 8^3 is read eight cubed. It may also be written $8 \times 8 \times 8$, and is equal to 512.

If the number to be raised to a given power is a fraction, both the numerator and denominator should be raised to the same power as indicated by the exponent; thus, $(\frac{11}{15})^2 = \dfrac{11 \times 11}{15 \times 15}$.

A little rule that is many times handy in finding the square of a mixed number if the fraction is $\frac{1}{2}$, as $2\frac{1}{2}$, $5\frac{1}{2}$, etc., is as follows:

Multiply the next smaller number by the next greater and add $\frac{1}{4}$. For example, $(3\frac{1}{2})^2$ is equal to 3 (the next smaller number) times 4 (the next larger number) $+ \frac{1}{4} = 12\frac{1}{4}$; $(10\frac{1}{2})^2$ is equal to $10 \times 11 + \frac{1}{4} = 110\frac{1}{4}$.

From equation No. 5, in E–2, it was stated that the amount of power lost in the electric circuit and which was dissipated as heat varied as the square of the current; this was expressed as $W = I^2 R$. Consider two conductors of same size and material and same length, one carrying 10 amperes, the other carrying 15 amperes; how do the relative amounts of lost power compare?

Solution: In the ratio of 10^2 to 15^2 read in the ratio of ten squared to fifteen squared, and this may be expressed as $\frac{10^2}{15^2} = \frac{100}{225}$, or as 4 to 9, $\therefore$ the conductor having a current of 15 amperes has $2\frac{1}{4}$ times as great a copper loss as the one carrying 10 amperes. To overcome this, a larger conductor should be used to reduce the resistance accordingly.

Evolution.

This is just the reverse of involution. It is the finding of the root (or extracting the root) of any number the power of which is given. Roots are named in a manner similar to the powers; thus, one of the two equal factors of a number is the second or square root; one of three equal roots is the third or cube root, etc.

The roots and powers of numbers are given in tables in many engineering hand-books, but it is extremely important that one understands how to compute both square and cube root. The diameter of a circle or the side of a square having a given area is obtained only by solving for the square root, and the side of a cube or the diameter of a sphere is obtained by solving for the cube root when the volume is known. Square root is also used a great deal in solving for the sides of triangles and in the applications of many formulas.

The radical sign, $\sqrt{}$, placed before a quantity indicates that some root is to be obtained. For example, $\sqrt{81}$, $\sqrt[3]{125}$, $\sqrt[4]{256}$ shows that the square root of 81, the cube root of 125, the fourth root of 256 are to be found. The small figures 3 and 4 are indexes showing what root is to be found if other than the square root.

These same operations may also be expressed by fractional exponents; as $81^{\frac{1}{2}}$, $125^{\frac{1}{3}}$, $256^{\frac{1}{4}}$. Many difficult problems in finding the roots of numbers are solved by logarithms, which will be explained later.

To extract the square root, begin at the decimal point and divide the number into periods of two figures each; thus 329.42567 would be divided 3′29.42′56′70. The operation of finding the square root is as shown in the following:

	3′29.42′56′70′00′00	18.15008+
	1	
20	229	
28	224	
360	542	
361	361	
3620	18156	
3625	18125	
36300	31700000	
363000	29040064	
3630000		
3630008		

As is shown in this computation, the whole number beginning at the decimal point may not have an even number of figures, so that the last period on the left may have only one. In the above number the figure 3 is the last period. The root will consist of as many figures as there are periods or divisions in the original number. The decimal point is located between the two figures corresponding to the two periods or divisions between which the decimal point occurs in the original number. In the above number the decimal point is located between the second and third periods, counting from the left, and is placed between the second and third figures in the root; thus $\sqrt{3'29.42'56'70} = 18.15008$. Ciphers may be added to the right of the decimal without changing its value, if it is desired to carry the root to a greater number of decimal places.

Explanation of the above Computation: The largest number which, when multiplied by itself, would be equal to or less than 3 is 1. This figure is the first figure of the root, and when multiplied by itself is placed under the first figure (3) from which it is subtracted. Then bring down the figures of the next period to the right, giving the number 229. The first figure (1) of the root is always multiplied by 2 and set down to the left of this new quantity (229). Affix a cipher to it, making it 20. This is now considered as a new trial divisor for the quantity 229. The next step is to find a number which, added to this, can be used as a multiplier of this sum and contained in 229; this is found to be 8. This is added to 20, making 28, and 8 times 28 are 224. Then 8 is the next figure in the root. As 29 is the last period in the original number before the decimal point, the decimal point in the root then follows this figure 8. Subtracting 224 from 229 leaves 5. Then bring down the next period, 42, forming the quantity 542. The right-hand figure of the last trial divisor, 28, is next added to this number; thus 28 plus 8 equals 36. Then affix a cipher to 36, making 360, which is the new trial divisor for the quantity 542. The process above mentioned is then repeated to find the next figure of the root, namely, to find a number which, added to this number 360, can be used as a multiplier of this sum and contained in 542. It is found to be 1. Then 361 is the divisor, and 1 times 361 is then subtracted from 542, giving 181. The period next in order is brought down, making 18156. The last divisor is then treated as above for a new trial divisor, namely, add the right-hand figure of the number 361 to 361, making 362, then affix a cipher, making 3620. Repeating the above process, the next figure of the root is found to be 5. This is sufficiently far to carry the root, when it comes so close, for any of the ordinary computations. If it is desired to carry it to more decimal places, ciphers may be added to the original number in periods of two each. When the last period was brought down after completing the operation for the figure 5, the quantity 3170 obtained and the trial divisor was 36300. As this is larger than the number 3170, a cipher is placed in the root after 5, a cipher is added to the trial divisor, making a new trial divisor, 363000, and the next period is brought down, forming the number 317000. The trial divisor is still larger, so that another cipher is added to the root, another to the trial divisor, and the next period is brought down, forming the number 31700000. The trial divisor 3630000 is now contained in this 8 times, which is the next figure of the root. The plus sign after the root indicates that it is not exact and that it may be carried further.

This same process is followed in finding the square root of a whole number. The decimal point should come after the figure corresponding to the last period of the number brought down. If the root is not exact, ciphers may be added after the decimal point of the number and treated the same as above.

In another paper, E-2, it was stated that the resistance of a conductor was dependent upon the area of its cross-section and that it was inversely proportional to the area of this cross-section. Then if the resistance of a given conductor and its area are known, we can compute the area for a similar conductor if its resistance is known, or the resistance if its area is known.

Example: If the resistance of a rod of copper one-half inch square is one ohm for a certain length, what would be the area and diameter of a similar bar having a resistance of three-fourths of an ohm, the length being the same?

Solution: It would obviously be larger in cross-section, as the resistance is to be less. By proportion, assuming that X is the dimension of one side of square rod, $(\frac{1}{2})^2 : X^2 :: \frac{3}{4} : 1$, since the resistance varies inversely as their areas. From a previous explanation of proportion,

$$\begin{aligned} X^2 &= (\tfrac{1}{2})^2 \times 1 \div \tfrac{3}{4} \\ &= \tfrac{1}{4} \times 1 \times \tfrac{4}{3} \\ &= \tfrac{4}{12} \\ &= \tfrac{1}{3} = \text{the area of the cross-section desired} \end{aligned}$$

in square inches. Then by extracting the square root of this, the dimension of one side may be found, $\frac{1}{3} = .3333+$ and $\sqrt{.3333} = .577''$.

Roots of fractions may be found by finding the given root of the numerator and denominator separately; for example, $\sqrt{\frac{9}{25}} = \frac{\sqrt{9}}{\sqrt{25}} = \frac{3}{5}$.

If this is expressed as a decimal, then $\sqrt{.36} = .6$. It is many times more convenient to extract the required root from a decimal than of the fraction, in which case it is obtained as shown in the problem described.

Cube Root: This is the process of finding the third power of a quantity and is a little more complicated, but it is not used as often as the square root. The process of finding this root is best explained by following an example. Let it be required to find the cube root of 329.42567, which would be written $\sqrt[3]{329.42567}$ or $329.42567^{\frac{1}{3}}$. The first operation is to point it off into periods of three figures each starting from the decimal point and pointing off in both directions.

	329.425′670′000 | 6.906+
$300 \times 6^2 = 10800$	216
$30 \times 6 \times 9 = 1620$	113425
$9^2 = 81)$	
12501)	112509
1701)	916670000
$300 \times 69^2 = 1428300$	
$300 \times 690^2 = 142830000$	
$30 \times 690 \times 6 = 124200$	
$6^2 = 36)$	
142954236)	857725416
124236)	58944584

Find the largest figure which when multiplied by itself three times does not exceed the left-hand period. It is seen that $7 \times 7 \times 7 = 343$, so that this is too large, and $6 \times 6 \times 6 = 216$, which is the first figure of the root. This is set down underneath the first period from which it is subtracted. The next period is then brought down, giving the number 113425 for the new dividend. To obtain the trial divisor, the square of the first figure of the root is multiplied by 300, giving 10800. It is seen that this will go into 113425 nine times. The trial divisor is then completed by adding 30 times the first figure and multiplied by the next figure in the root, or $30 \times 6 \times 9$, then add 9^2. The sum of these three quantities, 12501, is the complete trial divisor. This is then multiplied by 9, giving the number 112509, which is subtracted from 113425, with a remainder 916. The next period is then brought down, forming the number 916670. The first trial divisor for this quantity is obtained in same manner as preceding one; thus 300 times the square of the root so far found, or 300×69^2, equals 1428300. This may be more easily found after the first two figures are determined by bringing down the sum of 1620 and 81, which would be 1701 as used in the preceding divisor, then adding the three quantities connected by bracket and adding two ciphers. This obtains the same result as performing the squares and multiplying by 300. This gives a divisor larger than 916670, so that a cipher is placed in the root, making 6.90, three ciphers in the next period are brought down and two ciphers added to the new trial divisor, making 142830000. By inspection it is seen that this is contained in the new dividend 6 times, so that 6 is the next figure in the root; then repeat the same process for finding the complete divisor as previously. In adding the quantities for this divisor disregard the number 1428300, which was a trial divisor for a previous operation. This operation is the square of the figures found in the root 690 multiplied by 300, which gives 142830000, then add 30 times 690 times the new figure 6, which equals 124200, and also the square of this figure 6, which gives as the new divisor 142954236; this is the quantity which is multiplied by 6. If the operation was to be carried out further the next trial divisor would be the sum of the last three quantities in the bracket, to which should be added two ciphers.

Higher Roots: The fourth root is the square root of the square root; the sixth root is the cube root of the square root, or the square root of the cube root. Other roots are easily found by logarithms, which will be explained in another paper.

Application of Cube Root: A storage battery tank is in the form of a cube and holds 10 gallons of electrolyte when the plates are out; what is the inside depth of the tank?

Solution: From the tables, 1 gallon contains 231 cubic inches; then 10 gallons $= 2310$ cubic inches and $\sqrt[3]{2310} = 13.22 +$ inches.

Board and Timber Measure.

The unit of board and timber measure is the board foot, which is one foot square and one inch thick. In order to determine the number of board feet in a board or stick of timber, multiply the length in feet by the width in feet and the thickness in inches. If the width is in inches, divide the product by 12.

The volume in cubic feet of round timber is found by multiplying the length in feet by one-quarter of the mean diameter times the mean circumference or girth. By the "mean diameter" or "circumference" is meant the average, which in most cases would be that of the middle point of the log or pole. When the diameter and girth are given in inches, divide the product by 144. When the length, diameter and girth are all in inches, divide the product by 1728.

Problem: How many board feet shall be ordered to construct a false floor for a battery room 10 x 25 feet and the floor $1\frac{1}{2}$ inches thick? What would this cost at $25.00 per thousand feet?

Solution: Area of the floor is 10 times 25, or 250 sq. ft.; $1\frac{1}{2} \times 250 = 375$ board feet, since the thickness of the floor was $1\frac{1}{2}$ inches; $375 \div 1000 \times \$25.00 = \$9.38$.

Problem: How many cubic feet in a telephone pole 45 feet long, having a top diameter of 8 inches and the butt 16 inches?

Solution: Assuming that the pole is a straight taper, the mean or average diameter is $\frac{1}{2}(8 + 16)$, or 12 inches; then its girth at this point would be $12 \times 3.1416 = 37.7$ inches; then $\frac{45 \times 12 \times 37.7}{4 \times 144} = 35.34$ cu. ft.

The Circle.

This is a plane figure bounded by a curved line all points of which are equally distant from a point called the center. This curved boundary line is called the circumference. The radius is the straight line drawn from the center to the circumference. The diameter is the straight line drawn through the center and terminating at each end in the circumference. It is twice the length of the radius. The circumference is about $3\frac{1}{7}$ times as great as the diameter, or, more accurately, 3.1416. This relation is usually expressed as π, known as "pi." $2 \pi r =$ circumference.

Problem: A water tank measures 66 feet in circumference at the base. What will be the length of the longest beam extending across the bottom if it projects one foot on each end?

Solution: $66 \div 3.1416 = 21$ feet diameter; $21 + 2 = 23$ feet length of beam.

Problem: What is the diameter of a lead-covered cable, to the nearest sixteenth of an inch, if its circumference is 5 inches?

Solution: $5 \div 3.1416 = 1.59$ in. $\frac{59}{100} = \frac{X}{16}$ or $X = \frac{59 \times 16}{100} = 9.45$.

As this is nearer 9 than 10, this diameter would then be written $1\frac{9}{16}$.

The area of a circle is equal to the circumference times one-half the radius.

Since the circumference is equal to $2 \pi r$, the area may easily be expressed in terms of the radius; as, $2 \pi r \times \frac{r}{2}$ from previous statement. This is equal to $\pi r^2 = A$, the area.

The multiplication sign is many times omitted in writing formulas, but quantities written together as the above expressions are to be multiplied by each other; thus $2 \pi r$ is equal to $2 \times \pi \times r$.

The area may be also expressed in terms of the diameter. Since d, the diameter, is equal to $2r$ or $r = \frac{d}{2}$, we may substitute in the above equation $A = \pi r^2 = \pi \left(\frac{d}{2}\right)^2 = \frac{\pi d^2}{4}$. But $\pi = 3.1416$, then $\pi \div 4 = .7854$, and the formula for the area becomes $.7854\ d^2 = A$.

The diameter of the circle is also the length of one side of the square which would just enclose or circumscribe the circle. For rough estimating, the area of a circle is eight-tenths (.8) that of the circumscribing square. Then the area of the circumscribing square is approximately $1\frac{1}{4}$ times the area of the circle.

When the area of the circle is given and it is required to find its diameter, the following equation holds true:

$$d = \sqrt{\frac{A}{.7854}}$$

This is derived from the equation $A = .7854\ d^2$.

Problem: The area of the circular plot occupied by a turntable is 1257 sq. ft. What is the length of the track, assuming that it is the full length of the diameter?

Solution: $1257 \div .7854 = 1600$. $\sqrt{1600} = 40$ feet.

If a copper conductor having an area of 10382 cir. mils has resistance of 5.34 ohms per mile, what will be the diameter in mils of a similar wire having one-fourth the resistance?

Solution: Since the resistance is inversely proportional to the area, the lower resistance wire must be 4 times as great in area of cross-section, or $4 \times 10382 = 41528$. The diameter in mils is the square root of the area in circular mils. Then $\sqrt{41528} = 204$ mils.

From the wire tables these will be seen to agree very closely with the dimensions and resistances of No. 10 B. & S. and No. 4 B. & S. copper.

It might be stated here that the circle contains the greatest area of any plane figure for a given distance around or, circumference.

How does the circumference of a circle having an area of 500 sq. ft. compare with the circumference of a square plot with the same area?

Solution: $\pi\ r^2 = 500$ sq. ft., then $r^2 = \frac{500}{\pi} = 159.15$ sq. ft., $r = \sqrt{159.15} = 12.62$ ft. The circumference of this circle is then $2\,\pi\,12.62 = 72.294$ ft.

The circumference of the square having the same area is four times the length of one side $= 4\sqrt{500} = 4 \times .22.35 = 89.40$ ft.

The circumference of the above circle may be found from the equation $C = 3.545\ \sqrt{A}$, in which A is the area.

Concentric Circles.

Circles are said to be concentric when they have the same centers but different diameters or radii. The space between these concentric circles forms a ring, whose area is the difference between the areas of the greater and the smaller of the circles. Its width (not diameter) is the difference between their radii.

Problem: A turntable 40 ft. in diameter is surrounded by a cinder path 6 ft. wide. What is the area of the path?

Solution: The area of the turntable is 1257 sq. ft.; that of the outer circle is $\pi\ (20 + 6)^2 = 2123.7$; thus the difference between these areas is the area of the walk $= 2123.7 - 1257 = 866.7$ sq. ft.

Summary of the Formulas for the Circle.

D = diameter; R = Radius; C = Circumference; A = Area.

$$C = \pi\ D = 2\ \pi\ R = \frac{4A}{D} = 2\ \sqrt{\pi A} = 3.545\ \sqrt{A}.$$

$$D = C \div \pi = 0.31831C = 2\ \sqrt{\frac{A}{\pi}} = 1.12838\ \sqrt{A}.$$

$$R = C \div 2\ \pi = 0.159155C = \sqrt{\frac{A}{\pi}} = 0.564189\ \sqrt{A}.$$

$$A = D^2 \times .7854 = \tfrac{1}{2}\ CR = 4\ R^2 \times .7854 = \pi\ R^2 = \tfrac{1}{4}\ \pi\ D^2 = \tfrac{1}{4}\ CD = .07978\ C^2.$$

From these it will be seen that the circumference may be found direct if the diameter is given, if the radius is given, if the area and diameter are given, or if the area alone is given.

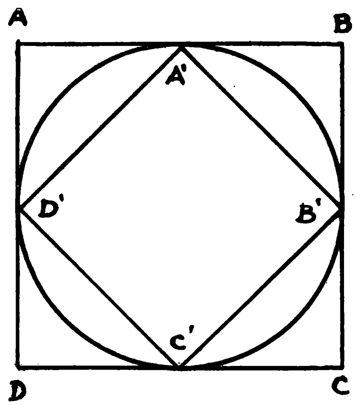
The diameter may be found direct if the circumference is given or if the area is given.

The radius may be found direct if the circumference is given or if the area is given.

The area may be found direct if the diameter is given, if the circumference and radius are given, if the radius is given, if the circumference and diameter are given, or if the circumference is given.

An inscribed square is one whose corners lie in the circumference of the circle; as, A′ B′ C′ D′.

A circumscribed square is one whose sides are tangent to the circle; as, A B C D. It is the square that will enclose the circle, whereas the inscribed square is the square enclosed by the circle.



Relation of Circle to the Square.

Side of an equal square	= Diameter of circle × .88623
Side of an equal square	= Circumference of circle × .28209
Perimeter (circumference) of an equal square	= Circumference of circle × 1.1284
Side of an inscribed square	= Diameter of circle × .7071
Side of an inscribed square	= Circumference of circle × .22508
Side of an inscribed square	= Area of circle × .90031 ÷ diameter
Area of circumscribed square	= Area of circle × 1.2732
Area of inscribed square	= Area of circle × .63662
Diameter of circumscribed circle	= Side of square × 1.4142
Circumference of circumscribed circle	= Side of square × 4.4428
Diameter of equal circle	= Side of square × 1.1284
Circumference of equal circle	= Side of square × 3.5449
Crcumference of equal circle	= Perimeter of square × .88623
Circular inches	= Square inches × 1.2732

Weights and Measures.

Long Measure—Measure of Length.

12	inches	= 1 foot
3	feet	= 1 yard
$5\frac{1}{2}$	yards, or $16\frac{1}{2}$ feet,	= 1 rod, pole, or perch
40	poles, or 220 yards,	= 1 furlong
8	furlongs, or 1760 yards, or 5280 feet,	= 1 mile
3	miles	= 1 league

Additional Measures of Length in Occasional Use: 1000 mils = 1 inch; 4 inches = 1 hand; 9 inches = 1 span; $2\frac{1}{2}$ feet = 1 military pace; 2 yards = 1 fathom.

Old Land Measure: 7.92 inches = 1 link; 100 links, or 66 feet, or 4 poles = 1 chain; 10 chains = 1 furlong; 8 furlongs = 1 mile, 10 square chains = 1 acre.

Square Measure—Measure of Surface.

144	square inches, or 183.35 circular inches,	= 1 square foot
9	square feet	= 1 square yard
$30\frac{1}{4}$	square yards, or $272\frac{1}{4}$ square feet,	= 1 square rod, pole, or perch
40	square poles	= 1 rood
4	roods, or 10 square chains, or 160 square poles, or 4840 square yards, or 43,560 square feet,	= 1 acre
640	acres	= 1 square mile

An acre equals a square whose side is 208.71 feet.

Circular Inch; Circular Mil: A circular inch is the area of a circle 1 inch in diameter = 0.7854 square inch.

1 square inch = 1.2732 circular inches.

A circular mil is the area of a circle 1 mil, or .001 inch in diameter, 1000^2 or 1,000,000 circular mils = 1 circular inch.

1 square inch = 1,273,239 circular mils.

The mil, and circular mil, are used in electrical calculations involving the diameter and area of wires.

Solid or Cubic Measure—Measure of Volume.

1728 cubic inches	= 1 cubic foot
27 cubic feet	= 1 cubic yard
1 cord of wood = a pile 4 × 4 × 8 feet	= 128 cubic feet
1 perch of masonry = $16\frac{1}{2} \times 1\frac{1}{2}$ × 1 foot	= $24\frac{3}{4}$ cubic feet

Liquid Measure.

4	gills	= 1 pint
2	pints	= 1 quart
4	quarts	= 1 gallon (U. S. 231 cubic inches, Eng. 277.274 cubic inches)
$31\frac{1}{2}$	gallons	= 1 barrel
42	gallons	= 1 tierce
2	barrels, or 63 gallons,	= 1 hogshead
84	gallons, or 2 tierces,	= 1 puncheon
2	hogsheads, or 126 gallons,	= 1 pipe or butt
2	pipes, or 3 puncheons,	= 1 tun

The U. S. gallon contains 231 cubic inches; 7.4805 gallons = 1 cubic foot. A cylinder 7 inches diameter and 6 inches high contains 1 gallon, very nearly, or 230.9 cubic inches. The British Imperial gallon contains 277.274 cubic inches = 1.20032 U. S. gallons.

The Miner's Inch: (Western U. S. for measuring flow of a stream of water).

The term Miner's Inch is more or less indefinite, for the reason that California water companies do not all use the same head above the center of the aperture, and the inch varies from 1.36 to 1.73 cubic feet, per minute, each; but the most common measurement is through an aperture 2 inches high and whatever length is required, and through a plank $1\frac{1}{4}$ inches thick. The lower edge of the aperture should be 2 inches above the bottom of the measuring box, and the plank 5 inches high above the aperture, thus making a 6-inch head above the center of the stream. Each square inch of this opening represents a miner's inch, which is equal to a flow of $1\frac{1}{2}$ cubic feet per minute.

Apothecaries' Fluid Measure.

60 minims	= 1 fluid drachm
8 drachms	= 1 fluid ounce

In the U. S. a fluid ounce is the 128th part of a U. S. gallon, or 1.805 cubic inches. It contains 456.3 grains of water at 39 degrees F. In Great Britain the fluid ounce is 1.732 cubic inches, and contains 1 ounce avoirdupois, or 437.5 grains of water at 62 degrees F.

Dry Measure, U. S.

2 pints	=	1 quart
8 quarts	=	1 peck
4 pecks	=	1 bushel

The standard U. S. bushel is the Winchester bushel, which is in cylinder form, 18½ inches diameter and 8 inches deep, and contains 2150.42 cubic inches.

A struck bushel contains 2150.42 cubic inches = 1.2445 cubic feet; 1 cubic foot = 0.80356 struck bushel. A heaped bushel is a cylinder 18½ inches diameter and 8 inches deep, with a heaped cone not less than 6 inches high. It is equal to 1¼ struck bushels.

The British imperial bushel is based on the imperial gallon, and contains 8 such gallons, or 2218.192 cubic inches = 1.2837 cubic feet. The English quarter = 8 Imperial bushels.

Capacity of a cylinder in U. S. gallons = square of diameter, in inches × height in inches × .0034. (Accurate within one part in 100,000.)

Capacity of a cylinder in U. S. bushels = square of diameter in inches × height in inches × .0003652.

MEASURES OF WEIGHT.

Avoirdupois or Commercial Weight.

16 drachms, or 437.5 grains,	=	1 ounce, oz.
16 ounces, or 7000 grains,	=	1 pound, lb.
28 pounds	=	1 quarter, qr.
4 quarters	=	1 hundredweight, cwt. = 112 lbs
20 hundredweight	=	1 ton of 2240 pounds, or long ton
2000 pounds	=	1 net, or short ton
2204.6 pounds	=	1 metric ton
1 stone	=	14 pounds
1 quintal	=	100 pounds

Troy Weight.

24 grains	=	1 pennyweight, dwt.
20 pennyweights	=	1 ounce, oz. = 480 grains
12 ounces	=	1 pound, lb. = 5760 grains

Troy weight is used for weighing gold and silver. The grain is the same in Avoirdupois, Troy and Apothecaries' weights. A carat, used in weighing diamonds, = 3.168 grains = .205 gramme.

Apothecaries' Weight.

20 grains	= 1 scruple
3 scruples	= 1 drachm = 60 grains
8 drachms	= 1 ounce = 480 grains
12 ounces	= 1 pound, lb. = 5760 grains

Circular Measure.

60 seconds	= 1 minute
60 minutes	= 1 degree
90 degrees	= 1 quadrant
360 degrees	= 1 circumference

Time.

60 seconds	= 1 minute
60 minutes	= 1 hour
24 hours	= 1 day
7 days	= 1 week

365 days, 5 hours, 48 minutes, 48 seconds = 1 year.

French and British (and American) Equivalent Measures.

FRENCH		BRITISH AND U. S.
1 metre	=	39.37 inches, or 3.28083 feet, or 1.09361 yards
0.3048 metre	=	1 foot
1 centimetre	=	0.3937 inch
2.54 centimetres	=	1 inch
1 millimetre	=	0.03937 inch, or $\frac{1}{25}$ inch, nearly
25.4 millimetres	=	1 inch
1 kilometre	=	1093.61 yards, or 0.62137 mile

Measure of Surface.

FRENCH.		BRITISH AND U. S.
1 square metre	=	10.764 square feet; 1.196 square yards
0.836 square metre	=	1 square yard
0.0929 square metre	=	1 square foot
1 square centimetre	=	0.155 square inch
6.452 square centimeters	=	1 square inch
1 square millimetre	=	0.00155 square inch
645.2 square millimetres	=	1 square inch
1 centiare = 1 square metre	=	10.764 square feet
1 are = 1 square decametre	=	1076.41 square feet
1 hectare = 100 ares	=	107641 square feet = 2.4711 acres
1 square kilometre	=	0.386109 square miles = 247.11 acres
1 square myriametre	=	38.6109 square miles

Of Volume.

FRENCH.		BRITISH AND U. S.
1 cubic metre	=	35.314 cubic feet 1.308 cubic yards
0.7645 cubic metre	=	1 cubic yard
0.02832 cubic metre	=	1 cubic foot
1 cubic decimetre	=	61.023 cubic inches 0.0353 cubic foot
28.32 cubic decimetres	=	1 cubic foot
1 cubic centimetre	=	0.061 cubic inch
16.387 cubic centimetres	=	1 cubic inch
1 cubic centimetre = 1 millilitre	=	0.061 cubic inch
1 centilitre	=	0.610 cubic inch
1 decilitre	=	6.102 cubic inches
1 litre = 1 cubic decimetre	=	61.023 cubic inches = 1.05671 quarts, U. S.
1 hectolitre or decistere	=	3.5314 cubic feet = 2.8375 bushels, U. S.
1 stere, kilolitre, or cubic metre	=	1.308 cubic yards = 28.37 bushels, U. S.

Of Capacity.

FRENCH.		BRITISH AND U. S.
1 litre (= 1 cubic decimetre)	=	61.023 cubic inches 0.03531 cubic foot 0.2642 gallon (American) 2.202 pounds of water at 62° F.
28.317 litres	=	1 cubic foot
4.543 litres	=	1 gallon (British)
3.785 litres	=	1 gallon (American)

Of Weight.

FRENCH.		BRITISH AND U. S.
1 gramme	=	15.432 grains
0.0648 gramme	=	1 grain
28.35 gramme	=	1 ounce avoirdupois
1 kilogramme	=	2.2046 pounds
0.4536 kilogramme	=	1 pound
1 tonne or metric ton 1000 kilogrammes	= =	0.9482 ton of 2240 lbs. 19.68 cwts. 2204.6 pounds
1.016 metric tons 1016 kilogrammes	=	1 ton of 2240 pounds

Weight of One Cubic Foot of Pure Water.

At 32	degrees F.	(freezing point)	62.418	lbs.
At 39.1	" "	(maximum density)	62.425	"
At 62	" "	(standard temperature)	62.355	"
At 212	" "	(boiling point, under 1 atmosphere)	59.76	"

American gallon = 231 cubic inches of water at 62 degrees F. = 8.3356 lbs.

British gallon = 277.274 cubic inches of water at 62 degrees F. = 10 lbs.

QUESTIONS.

1.—Reduce to a common denominator the fractions $\frac{3}{15}$, $\frac{3}{10}$, $\frac{1}{2}$.

2.—Reduce to lowest terms the following fractions: (*a*) $\frac{9}{12}$, (*b*) $\frac{10}{15}$, (*c*) $\frac{4}{20}$, (*d*) $\frac{6}{10}$.

3.—Change to mixed numbers the fractions (*a*) $\frac{40}{7}$, (*b*) $\frac{35}{10}$, (*c*) $\frac{15}{8}$, (*d*) $\frac{20}{3}$.

4.—Change to improper fractions the mixed numbers (*a*) $3\frac{1}{2}$, (*b*) $5\frac{2}{3}$, (*c*) $7\frac{8}{10}$, (*d*) $2\frac{7}{12}$, (*e*) $4\frac{1}{2}$.

5.—Add the fractions (*a*) $\frac{7}{15}$, $\frac{3}{10}$, $\frac{1}{2}$; (*b*) $\frac{5}{12}$, $3\frac{1}{3}$, $\frac{5}{8}$.

6.—Subtract the following: (*a*) $\frac{8}{10} - \frac{3}{5}$; (*b*) $\frac{3}{5} - \frac{3}{10}$; (*c*) $\frac{1}{2} - \frac{1}{3}$; (*d*) $3\frac{1}{2} - 2\frac{1}{3}$; (*e*) $3\frac{1}{2} - 1\frac{3}{4}$.

7.—Multiply (*a*) $\frac{3}{5} \times \frac{1}{4} \times \frac{7}{10}$; (*b*) $\frac{1}{2} \times \frac{1}{3} \times \frac{9}{11}$; (*c*) $\frac{1}{5} \times 1\frac{3}{7} \times \frac{11}{20}$.

8.—Divide (*a*) $\frac{1}{4} \div \frac{1}{2}$; (*b*) $3 \div \frac{1}{7}$; (*c*) $14\frac{1}{2} \div \frac{9}{11}$; (*d*) $\frac{4}{13} \div 4\frac{1}{3}$.

9.—Write out in words the following: (*a*) 13.405, (*b*) 3.0026, (*c*) 25.49013.

10.—Add the following: (*a*) 17.94, 1.002, .138, 26.04; (*b*) 23.17, 102.708, 16.001.

11.—Subtract (*a*) 16.703 — 10.94; (*b*) 9.086 — 3.1; (*c*) 27.060 — 0.49; (*d*) 15.013 — 14.082.

12.—Reduce the following to decimals having same denominator: (*a*) .04; (*b*) .9; (*c*) .385; (*d*) .0981.

13.—Change to common fraction and reduce to lowest terms (*a*) .125; (*b*) .25; (*c*) .375; (*d*) .20; (*e*) .625.

14.—Change to equivalent decimals (*a*) $\frac{1}{5}$; (*b*) $\frac{1}{4}$; (*c*) $\frac{1}{3}$; (*d*) $\frac{1}{2}$; (*e*) $\frac{4}{5}$; (*f*) $\frac{8}{10}$; (*g*) $\frac{3}{50}$.

15.—Multiply (*a*) 6.842×5.3; (*b*) $52.731 \times .004$; (*c*) $26 \times .125$.

16.—Divide (*a*) $2.68 \div 14.2$; (*b*) $25.06 \div 50.12$; (*c*) $1248.3 \div 29.4$; (*d*) $1098.10 \div 1.26$.

17.—What is a ratio? Distinguish between arithmetical and geometrical ratio.

18.—How does inverse ratio differ from direct ratio?

19.—If the resistance of two similar conductors varies inversely as the square of their diameters, how does the resistance of a rod 460 mils in diameter compare with one having a diameter of 162 mils?

20.—If 15 men can erect a certain piece of line in 36 days, in what time can they perform the same work with 9 additional men in the force, assuming that all are equally as efficient as the fifteen?

21.—The top of a wire fixture on a bridge casts a shadow 96 ft. from a point immediately underneath and measured along the track, at the same time a 10-ft. pole in a vertical position makes a shadow 15 ft. long. How high is the fixture above the track?

22.—Raise the following numbers to the powers given: (*a*) 2^2, (*b*) 5^2, (*c*) 12^2, (*d*) 4^3, (*e*) 8^3, (*f*) 5^5, (*g*) $(3\frac{1}{2})^2$, (*h*) $(5\frac{1}{2})^2$, (*i*) $(8\frac{1}{3})^2$.

23.—Find the square root of the following: (*a*) 169, (*b*) 400, (*c*) 132, (*d*) 3, (*e*) 1.025, (*f*) 16.01.

24.—Find the cube root of (*a*) 614.125, (*b*) 1000.00, (*c*) 373.248.

25.—How many litres in a tank in the shape of a cube 3 ft. on a side?

26.—A walk is made of boards 6 in. wide, $1\frac{1}{4}$ in. thick, and the boards are laid with $\frac{1}{2}$-in. space between them. The walk is 3 ft. wide, but the boards are 10 ft. long. If the walk is 500 ft. long and the boards cost $25.00 per M. ft., how many board ft. should be ordered and what will it cost?

27.—Assuming that chestnut poles weigh 55 lbs. per cubic ft., what will be the weight of a pole 40 ft. long having top diameter of 8 in., butt diameter $13\frac{1}{2}$ in.?

28.—What should be the length of the rod to form a hoop for a circular water-tank at a point where the outside diameter is 12.4 ft., if the hoop should lap 4 in. to make a good joint and there are 4 joints in the hoop?

29.—If this is the lower hoop and comes over the joint for the bottom, and the thickness of the boards in the side is 2 in., what is the inside area of the bottom?

30.—What is the area of a square platform that will just come flush with the bottom of this tank?

31.—If this tank had been square and held the same number of gallons, what would be the circumference, assuming that the height is the same as the circular tank?