PAMPHLET M–1–B

ELEMENTARY ARITHMETIC

COMMON FRACTIONS
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FRACTIONS.

A fraction is one or more of the equal parts of a unit; for example, \(\frac{7}{8}, \frac{3}{4}\). Thus: take the fraction \(\frac{3}{4}\) inch, which means that 1 inch is divided into 4 equal parts, each part being \(\frac{1}{4}\) of an inch in length, and that 3 of these parts are taken. When it is said that an iron bar is \(\frac{3}{4}\) inch thick, it means that the inch is divided into 4 equal parts, and that the bar is as thick as 3 of these parts. The 3 is the numerator and the 4 the denominator.

The terms of a fraction are the numerator and denominator.

The denominator of a fraction shows the number of equal parts into which the unit is divided. It is written below the line.

The numerator of a fraction shows how many of these parts are taken. It is written above the line.

The line between the numerator and denominator indicates division, and means divided by. Thus: \(\frac{2}{3}\) means that 2 is divided by 3 and is equivalent to \(2 \div 3\).

A proper fraction is a fraction whose numerator is less than its denominator; for example, \(\frac{1}{2}, \frac{4}{5}, \frac{6}{7}, \frac{7}{8}\).

An improper fraction is a fraction whose numerator is equal to or greater than its denominator; for example, \(\frac{4}{4}, \frac{5}{6}\).

A mixed number is a number expressed by a whole number, or integer, and a fraction. Thus: \(6\frac{2}{3}\) is a mixed number, and is read six and two-thirds.

A complex fraction is a fraction in which either the numerator, or the denominator, or both are fractions. Thus: \(\frac{\frac{2}{1}}{\frac{2}{3}}\) which is read two divided by one-half; and \(\frac{\frac{3}{4}}{\frac{5}{4}}\) which is read two-thirds divided by four-fifths, are complex fractions.

Whenever a heavy line is used, as shown in these complex fractions, it means that all above this line is to be divided by all below it.

(3)
READING AND WRITING FRACTIONS.

A fraction is read by reading the numerator (the number of parts) and then the denominator (the size of parts). Thus: \( \frac{4}{5} \) is read four-fifths.

- \( \frac{1}{2} \) is read one-half.
- \( \frac{2}{3} \) is read two-thirds.
- \( \frac{5}{6} \) is read five-sixths.
- \( \frac{11}{12} \) is read eleven-twelfths.
- \( \frac{27}{45} \) is read twenty-seven forty-fifths.

A fraction is written by placing the numerator (the number of parts) above the line, and the denominator (the size of parts) below the line. Thus: three-fifths is written \( \frac{3}{5} \).

- Four-sevenths is written \( \frac{4}{7} \).
- Six-eighths is written \( \frac{6}{8} \).
- Seven-elevenths is written \( \frac{7}{11} \).
- Eleven-fifteenths is written \( \frac{11}{15} \).
- Thirteen-twentieths is written \( \frac{13}{20} \).

REDUCTION OF FRACTIONS.

Reduction of fractions is the process of changing their form without changing their value.

To reduce a fraction to higher terms.

Rule 1:—Multiply both the numerator and the denominator of the given fraction by a number which is found by dividing the desired denominator by the denominator of the given fraction.
Example:

Reduce $\frac{3}{4}$ to an equivalent fraction having 8 for its denominator.

Solution:

8, the desired denominator, divided by 4, the given denominator = 2. Multiply both numerator and denominator by 2, as follows: $\frac{3 \times 2}{4 \times 2} = \frac{6}{8}$. The value is not changed because $\frac{3}{4} = \frac{6}{8}$, as shown in Fig. 1.

\[
\begin{array}{cccccc}
& & & & & \\
& 1 & & 3 & & 5 & & 7 & & \\
\hline
\frac{1}{8} & & \frac{3}{8} & & \frac{5}{8} & & \frac{7}{8} & & \\
\hline
\frac{2}{8} & & \frac{4}{8} & & \frac{6}{8} & & \frac{8}{8} & & \\
\frac{1}{4} & & \frac{2}{4} & & \frac{3}{4} & & \frac{4}{4} & & \\
\frac{1}{2} & & & & & & & & \\
\end{array}
\]

Fig. 1.

To reduce a fraction to lower terms.

Rule 2:—Divide both numerator and denominator by the same number.

Example:

Reduce $\frac{4}{8}$ to lower terms.

Solution:

$\frac{4}{8}$ is reduced to lower terms by dividing both numerator and denominator by 2, as follows: $\frac{4 \div 2}{8 \div 2} = \frac{2}{4}$. The value is not changed because $\frac{4}{8} = \frac{2}{4}$ as shown in Fig. 1.
A fraction is **reduced to its lowest terms** when both its numerator and denominator cannot be divided by the same number without a remainder. Thus: each of the fractions $\frac{1}{4}, \frac{4}{5}, \frac{6}{7}, \frac{7}{8}$ is reduced to its lowest terms, for no number except 1 will exactly divide both numerator and denominator of any one of them.

**Example:**

Reduce $\frac{4}{32}$ to its lowest terms.

**Solution:**

Since dividing both terms of a fraction by the same number does not change its value, divide each term by 4, as this is the largest number that is contained in both the numerator and the denominator without a remainder. $\frac{4}{32} \div 4 = \frac{1}{8}$, which fraction is in its lowest terms, since neither of the terms in the fraction, $\frac{1}{8}$, can be divided by the same number, except 1, without a remainder.

A **common divisor** of two or more numbers is a number that exactly divides each of them. Thus: 5 is a **common divisor** of 15 and 20.

**Similar fractions** are fractions that have the same denominator. Thus: $\frac{2}{5}$ and $\frac{3}{5}$ are similar fractions, as both have the same denominator, 5.

**Dissimilar fractions** are fractions that do not have the same denominator. Thus: $\frac{2}{3}$ and $\frac{4}{5}$ are dissimilar fractions, as one has the denominator 3, and the other, 5.

A **common denominator** of two or more fractions is a number which is exactly divisible by the denominators of each fraction. Thus: 50 is a common denominator of $\frac{3}{10}$ and $\frac{4}{5}$, because 50 is divisible by each of the denominators, 10 and 5, without a remainder.

The **least common denominator** (abbreviation L. C. D.) of two or more fractions is the smallest number that is exactly divisible by the denominators of each fraction. Thus: 10 is the least common denominator of $\frac{3}{10}$ and $\frac{4}{5}$, because 10 is the smallest number that is divisible by each of the denominators, 10 and 5, without a remainder.
To reduce fractions to equivalent fractions having a common denominator.

Rule 3:--Multiply both terms of each fraction by the denominators of each of the other fractions.

Example:

Reduce \(\frac{3}{4}, \frac{1}{2}, \frac{3}{5}\) to fractions having a common denominator.

Solution:

A common denominator of these fractions is a number that will exactly contain 4, 2, and 5, and is \(4 \times 2 \times 5 = 40\). Multiply both terms of \(\frac{3}{4}\) by 2 and 5 (the denominators of the other fractions), as follows: \(\frac{3 \times 2 \times 5}{4 \times 2 \times 5} = \frac{30}{40}\); likewise multiply both terms of \(\frac{1}{2}\) by 4 and 5 (the denominators of the other fractions), \(\frac{1 \times 4 \times 5}{2 \times 4 \times 5} = \frac{20}{40}\); and both terms of \(\frac{3}{5}\) by 4 and 2 (the denominators of the other fractions), \(\frac{3 \times 4 \times 2}{5 \times 4 \times 2} = \frac{24}{40}\).

Therefore, \(\frac{3}{4} = \frac{30}{40}, \frac{1}{2} = \frac{20}{40}, \frac{3}{5} = \frac{24}{40}\).

Example:

Find the least common denominator of \(\frac{3}{4}, \frac{5}{8}, \frac{4}{9}\), and \(\frac{1}{24}\).

Solution:

\[\begin{array}{c}
2)4\ 8\ 9\ 24 \\
2)2\ 4\ 9\ 12 \\
2)1\ 2\ 9\ 6 \\
3)1\ 1\ 9\ 3 \\
\hline
1\ 1\ 3\ 1
\end{array}\]

The least common denominator is \(2 \times 2 \times 2 \times 3 \times 3 = 72\).

Explanation: Place the denominators of the fractions in a row, separating each by a space. Draw a line to the left of the first number, and below all of the numbers. Divide them by any prime number that is contained into at least two of them without a remainder, and bring down those numbers to the row below which will not contain the divisor without a remainder. One of the prime numbers which is contained into at least two of them without a remainder is 2. Divide by 2, and the result is 2, 4, 9, 12. Divide the second row of numbers by 2, and the result
is 1, 2, 9, 6. Divide the third row by 2, and the result is 1, 1, 9, 3. Divide the fourth row by 3, and the result is 1, 1, 3, 1. Therefore, the least common denominator, L. C. D., in this example is $2 \times 2 \times 2 \times 3 \times 3 = 72$.

Note:—In any problem this process is continued until no prime number, except 1, will exactly divide more than one of the remaining numbers. The Least Common Denominator is, then, the product of all the divisors and the numbers remaining in the last line.

To reduce whole or mixed numbers to improper fractions.

Rule 4:—Multiply the whole number by the denominator of the fraction, add the numerator to the product, and write the denominator under the sum.

Example:
Reduce 8 to an improper fraction having 4 as a denominator.

Solution:
In 1 there are 4 fourths, and in 8 there are $8 \times 4$ fourths, or $\frac{32}{4}$.

This may be written $8 \times \frac{4}{4} = \frac{32}{4}$.

Example:
Reduce $\frac{5}{3}$ to an improper fraction.

Solution:
In 1 there are 3 thirds, and in 5 there are $5 \times 3$ thirds = 15 thirds, $\frac{15}{3}$, which, added to the $\frac{2}{3}$, make 17 thirds, or $\frac{17}{3}$.

Or: $5 \times \frac{3}{3} = \frac{15}{3}$ and $\frac{15}{3} + \frac{2}{3} = \frac{17}{3}$.

This may be written $\left(\frac{3}{3} \times 5\right) + \frac{2}{3} = \frac{15}{3} + \frac{2}{3} = \frac{17}{3}$.

To reduce an improper fraction to a whole or mixed number.

Rule 5:—Divide the numerator by the denominator; the quotient will be the whole number, and the remainder, if there be any, will be the numerator of the fraction having the same denominator as the improper fraction.
Example:
Reduce \( \frac{64}{16} \) to a whole or mixed number.

Solution:
16 is contained in 64, 4 times without a remainder. Therefore, \( \frac{64}{16} = 4 \).

Example:
Reduce \( \frac{17}{2} \) to a mixed number.

Solution:
2 is contained in 17, 8 times with a remainder of 1. As this 1 is also divided by 2, its value is \( \frac{1}{2} \). Therefore, \( \frac{17}{2} = 8 + \frac{1}{2} = 8\frac{1}{2} \).

Example:
Reduce \( \frac{35}{14} \) to a mixed number.

Solution:
\[ 14) \begin{array}{c|c|c} 35 & 2 \\ 14 & 1 \\ 28 & 1 \\ 7 & 2 \\ 7 & 1 \\ 14 & 2 \end{array} \]

Therefore \( \frac{35}{14} = 2\frac{1}{2} \).

Note:—It is always customary to reduce the fractional part of the mixed number to its lowest terms, as shown.

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**ADDITION OF FRACTIONS.**

Addition of fractions is the process of finding the sum of two or more fractions.

To add fractions.

Rule 6:—Reduce the fractions to equivalent fractions having a common denominator; then add their numerators, and place this sum over the common denominator.
A common fraction denotes that a thing has been divided into a number of equal parts; thus, if 1 inch is divided into 8 equal parts, and 3 of these parts are taken, this will make \( \frac{3}{8} \) of an inch; or if it is divided into 16 equal parts, and 7 of these parts are taken, this will make \( \frac{7}{16} \) of an inch. Now, in order to add fractions, their denominators, which represent the number of equal parts into which the thing is divided, must be of the same size. \( \frac{1}{4} \) of a foot cannot be added to \( \frac{1}{2} \) of a foot before first finding out how many fourths there are in \( \frac{1}{2} \), because the denominator denotes the number of equal parts into which the unit is divided, which, in the first case, is 4 equal parts, and in the second case, 2 equal parts. These parts are of unequal size and cannot be added until reduced to a common denominator. In \( \frac{1}{2} \) there are \( \frac{2}{4} \).

Therefore, \( \frac{1}{4} \) of a foot + \( \frac{2}{4} \) of a foot equals \( \frac{3}{4} \) of a foot.

Similarly, in order to add \( \frac{1}{2} \) of an inch to \( \frac{3}{4} \) of an inch, it is necessary to find out how many fourths there are in \( \frac{1}{2} \). In \( \frac{1}{2} \) there are \( \frac{2}{4} \).

Therefore, \( \frac{2}{4} \) in. + \( \frac{3}{4} \) in. = \( \frac{5}{4} \) in. or 1 \( \frac{1}{4} \) inches.

Example:

\[
\frac{4}{5} + \frac{8}{9}.
\]

Solution:

The common denominator is the product of their denominators, or \( 9 \times 5 = 45 \). Hence, \( \frac{4 \times 9}{5 \times 9} = \frac{36}{45} \) and \( \frac{8 \times 5}{9 \times 5} = \frac{40}{45} \). Therefore, \( \frac{36}{45} + \frac{40}{45} = \frac{76}{45} = 1 \frac{31}{45} \).

To add mixed numbers.

Rule 7:—Add the whole numbers and fractions separately; then add the results together.
Example:

\[ 15 \frac{1}{4} + 20 \frac{1}{2} \]

Solution:

\[ 15 + 20 = 35, \text{ and } \frac{1}{4} + \frac{1}{2} = \frac{3}{4} \]

Therefore, \( 15 \frac{1}{4} + 20 \frac{1}{2} = 35 + \frac{3}{4} = 35\frac{3}{4} \).

This operation may be written:

\[ 15 \frac{1}{4} + 20 \frac{1}{2} = 35 \frac{3}{4} \]

In case the sum of the fractions is equal to or greater than 1, add to the sum of the whole numbers, the number of units made by the fractions.

Example:

\[ 20 \frac{7}{8} + 23 \frac{3}{4} \]

Solution:

\[ 20 + 23 = 43, \text{ and } \frac{7}{8} + \frac{3}{4} = 7 + \frac{6}{8} = \frac{13}{8} = 1 \frac{5}{8} \]

Therefore, \( 20 \frac{7}{8} + 23 \frac{3}{4} = 43 + 1 \frac{5}{8} = 44 \frac{5}{8} \).

This operation may be written:

\[ 20 \frac{7}{8} + 23 \frac{3}{4} = 44 \frac{5}{8} \]

Example:

What is the total length of two pieces of pipe \( 27 \frac{5}{8} \) inches and \( 23 \frac{1}{2} \) inches long respectively?
Solution:

\[ 27 + 23 = 50, \text{ and } \frac{5}{8} + \frac{1}{2} = \frac{9}{8} = 1 \frac{1}{8}. \]

Therefore, \( \frac{27}{8} + 23 \frac{1}{2} = 50 + 1 \frac{1}{8} = 51 \frac{1}{8} \), the total length in inches.

This operation may be written \( 27 \frac{5}{8} = 27 \frac{5}{8} \)
\[ 23 \frac{1}{2} = 23 \frac{4}{8} \]
\[ 50 \frac{9}{8} = 51 \frac{1}{8} \]

Example:

How long must a piece of iron be to cut from it four pieces, \( 4 \frac{1}{2} '', 2 \frac{3}{4} '', 11 \frac{2}{3} '', \) and \( 15 \frac{5}{6} '' \) long respectively? (The sign " denotes inches.)

Solution:

Add the whole numbers, \( 4 + 2 + 11 + 15 = 32 \). To add the fractions, first find their least common denominator; thus,
\[ 2 \) 2 4 3 6 \]
\[ 3) 1 2 3 3 \]
\[ 1 2 1 1 \]

Their least common denominator is then \( 2 \times 3 \times 2 = 12 \).

Then, \( \frac{1}{2} = \frac{6}{12} \), \( \frac{3}{4} = \frac{9}{12} \), \( \frac{2}{3} = \frac{8}{12} \), \( \frac{5}{6} = \frac{10}{12} \).

\[ \frac{6}{12} + \frac{9}{12} + \frac{8}{12} + \frac{10}{12} = \frac{33}{12} = \frac{9}{12} = \frac{2}{4} \text{ or } \frac{3}{4}. \]

Therefore, \( 4 \frac{1}{2} '' + 2 \frac{3}{4} '' + 11 \frac{2}{3} '' + 15 \frac{5}{6} '' = 32 '' + 2 \frac{3}{4} '' = 34 \frac{3}{4} '', \text{ the length of the piece necessary.} \)

**SUBTRACTION OF FRACTIONS.**

**Subtraction of fractions** is the process of finding the difference between two fractions.

**To subtract fractions.**

Rule 8:—Reduce the fractions to a common denominator; then take the difference between the numerators, and place the result over the common denominator.
Example:

Subtract $\frac{3}{4}$ from $\frac{4}{5}$. This problem may also be written $\frac{4}{5} - \frac{3}{4}$.

Solution:

Reduce the fractions to fractions having the same denominator, $\frac{4 \times 4}{5 \times 4} = \frac{16}{20}$ and $\frac{3 \times 5}{4 \times 5} = \frac{15}{20}$.

Therefore, $\frac{4}{5} - \frac{3}{4} = \frac{16}{20} - \frac{15}{20} = \frac{1}{20}$.

**To subtract mixed numbers.**

Rule 9:—Subtract the whole numbers and fractions separately; then take the sum of the remainders.

Example:

What is the difference between $15\frac{6}{7}$ and $13\frac{2}{3}$? This problem may also be written $15\frac{6}{7} - 13\frac{2}{3}$.

Solution:

$15 - 13 = 2$, and $\frac{6}{7} - \frac{2}{3} = \frac{18}{21} - \frac{14}{21} = \frac{4}{21}$.

Therefore, $15\frac{6}{7} - 13\frac{2}{3} = 2 + \frac{4}{21} = \frac{2}{21} + \frac{4}{21} = \frac{6}{21}$.

This operation may be written $15\frac{6}{7} = 15\frac{18}{21}$

$13\frac{2}{3} = 13\frac{14}{21}$

$\frac{6}{21} + \frac{14}{21} = \frac{20}{21}$.

Example:

What is the difference in the length of two pieces of conduit $23\frac{1}{4}$" and $17\frac{7}{8}$" long respectively?

Solution:

$\frac{1}{4} = \frac{2}{8}$. In this case, $\frac{7}{8}$ cannot be subtracted from $\frac{2}{8}$; so $1$ is taken from $23$, leaving $22$, and added to the fraction $\frac{2}{8}$, making it $\frac{10}{8}$.

$23\frac{1}{4}$ may then be written $22\frac{10}{8}$.
Subtract the fractions; thus, \( \frac{10}{8} - \frac{7}{8} = \frac{3}{8} \). Then subtract the whole numbers; thus, \( 22 - 17 = 5 \).

Therefore, \( 23\frac{1}{4} - 17\frac{7}{8} = 5 + \frac{3}{8} = 5\frac{3}{8} \).

This operation may be written \( 23\frac{1}{4} = 22\frac{10}{8} \), \( 17\frac{7}{8} = 17\frac{7}{8} \), and \( 5\frac{3}{8} \).

Example:

From a piece of wire \( 28\frac{3}{4} \) long, a piece \( 17\frac{3}{8} \) is cut. How long is the remaining piece?

Solution:

\[
\frac{3}{4} = \frac{6}{8}. \quad \text{Then} \quad \frac{6}{8} - \frac{3}{8} = \frac{3}{8}, \quad \text{and} \quad 28'' - 17'' = 11''.
\]

Therefore, \( 28\frac{3}{4}'' - 17\frac{3}{8}'' = 11'' + \frac{3}{8}'' = 11\frac{3}{8}'' \).

This operation may be written \( 28\frac{3}{4} = 28\frac{6}{8} \), \( 17\frac{3}{8} = 17\frac{3}{8} \), and \( 11\frac{3}{8} \).

Note:—Mixed numbers may also be subtracted by reducing them to improper fractions and proceeding as in Rule 8.

Example:

\[
40\frac{1}{3} - 14\frac{1}{7}.
\]

Solution:

\[
40\frac{1}{3} = \frac{121}{3}, \quad \text{and} \quad 14\frac{1}{7} = \frac{99}{7}. \quad \text{The common denominator is 21; then} \quad \frac{121}{3} = \frac{121 \times 7}{3 \times 7} = \frac{847}{21}, \quad \text{and} \quad \frac{99}{7} = \frac{99 \times 3}{7 \times 3} = \frac{297}{21}.
\]

Therefore, \( 40\frac{1}{3} - 14\frac{1}{7} = \frac{847}{21} - \frac{297}{21} = \frac{550}{21} = 26\frac{4}{21} \).
MULTIPLICATION OF FRACTIONS.

Multiplication of fractions is the process of finding the product of two or more fractions.

To multiply a fraction by a fraction.

Rule 10:—Multiply the numerators together to obtain the numerator of the product, and multiply the denominators together to obtain the denominator of the product.

Example:

Find the product of \(\frac{7}{8} \times \frac{5}{16}\).

Solution:

The product of the numerators is \(7 \times 5 = 35\), and the product of the denominators is \(8 \times 16 = 128\).

Therefore, \(\frac{7}{8} \times \frac{5}{16} = \frac{35}{128}\).

Note:—The word of between fractions or between a fraction and a whole number means the same as the multiplication sign.

Example:

\(\frac{4}{5}\) of \(\frac{2}{3}\) = \(\frac{4}{5} \times \frac{2}{3} = \frac{8}{15}\).

Example:

\(\frac{2}{3}\) of \(\frac{12}{5}\) = \(\frac{2}{3} \times \frac{12}{5} = \frac{24}{15} = \frac{9}{15} = \frac{3}{5}\).

To multiply a fraction by a whole number.

Rule 11:—Multiply the numerator of the fraction by the whole number, and place the product thus obtained over the denominator.

Example:

Multiply \(\frac{5}{6}\) by 7.

Solution:

Multiply the numerator of the fraction by the whole number; thus, \(\frac{5}{6} \times 7 = \frac{35}{6}\) or \(5\frac{5}{6}\).

Example:

What will be the total length of 9 pieces of tool steel each \(\frac{7}{8}\) of a foot long?
Solution:

The total length of the pieces may be found by multiplying the numerator of the fraction by the whole number; thus,

\[ 9 \times \frac{7}{8} \text{ ft.} = \frac{63}{8} \text{ ft. or } 7 \frac{7}{8} \text{ feet}. \]

Example:

How many feet in \( \frac{1}{2} \) mile?

Solution:

There are 5280 feet in 1 mile, and in \( \frac{1}{2} \) mile there are

\[ \frac{1}{2} \text{ of } 5280 \text{ ft.} = \frac{5280}{2} \text{ ft.} = 2640 \text{ feet.} \]

To multiply a mixed number by a whole number.

Rule 12:—Multiply the whole numbers together; then multiply the fraction of the mixed number by the whole number, and add the products thus obtained.

Example:

Multiply \( 23 \frac{1}{4} \) by 3.

Solution:

\[ 23 \times 3 = 69, \text{ and } \frac{1}{4} \times 3 = \frac{3}{4}. \]

Therefore, \( 23 \frac{1}{4} \times 3 = 69 + \frac{3}{4} = 69 \frac{3}{4} \).

This operation may be written

\[ \frac{3}{69 \frac{3}{4}} \]

Example:

\( 31 \frac{3}{4} \times 3 \).

Solution:

\[ 31 \times 3 = 93, \text{ and } \frac{3}{4} \times 3 = \frac{9}{4} = 2 \frac{1}{4}. \]

Therefore, \( 31 \frac{3}{4} \times 3 = 93 + 2 \frac{1}{4} = 95 \frac{1}{4} \).
This operation may be written $31 \frac{3}{4}$

\[
\begin{array}{c}
3 \\
93 \\
\hline
2 \frac{1}{4} \\
95 \frac{1}{4}
\end{array}
\]

Example:

If wire nails cost 9 cents per pound, what will $11 \frac{1}{4}$ pounds cost?

Solution:

\[
11 \times 9 \text{ cents} = 99 \text{ cents}, \quad \text{and} \quad \frac{1}{4} \times 9 \text{ cents} = \frac{9}{4} \text{ cents}
\]

\[
= 2 \frac{1}{4} \text{ cents}.
\]

Therefore, $11 \frac{1}{4} \times 9 \text{ cents} = 99 \text{ cents} + 2 \frac{1}{4} \text{ cents} = 101 \frac{1}{4} \text{ cents}$, or $\$1.01 \frac{1}{4}$.

This operation may be written $11 \frac{1}{4}$

\[
\begin{array}{c}
9 \\
99 \\
\hline
2 \frac{1}{4} \\
101 \frac{1}{4}
\end{array}
\]

To multiply one mixed number by another mixed number.

Rule 13:—Reduce both mixed numbers to improper fractions and proceed as when multiplying a fraction by a fraction.

Example:

Multiply $3 \frac{1}{2}$ by $5 \frac{3}{5}$.

Solution:

\[
3 \frac{1}{2} = \frac{7}{2}, \quad \text{and} \quad 5 \frac{3}{5} = \frac{28}{5}.
\]

Therefore, $3 \frac{1}{2} \times 5 \frac{3}{5} = \frac{7}{2} \times \frac{28}{5} = \frac{196}{10} = 19 \frac{6}{10} \text{ or } 19 \frac{3}{5}$. 

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DIVISION OF FRACTIONS.

Division of fractions is the process of finding the quotient of two fractions.

To divide a fraction by a fraction.

Rule 14: Invert the divisor; then multiply the numerators together for the numerator of the quotient, and the denominators together for the denominator of the quotient.

To invert a fraction is to turn it upside down; thus, $\frac{3}{4}$ inverted is $\frac{4}{5}$, $\frac{5}{6}$ inverted is $\frac{6}{5}$, $\frac{1}{5}$ inverted is $\frac{1}{5}$, and $\frac{1}{5}$ inverted is $\frac{5}{1}$.

When a whole number is inverted it becomes the denominator of a fraction with 1 for its numerator; thus, 4 inverted becomes $\frac{1}{4}$, 12 inverted becomes $\frac{1}{12}$, etc.

A whole number, when standing alone, is always considered as having 1 for its denominator; thus, $4 = \frac{4}{1}$, $12 = \frac{12}{1}$, etc.

Example:

Divide $\frac{4}{5}$ by $\frac{8}{9}$.

Solution:

The divisor, $\frac{8}{9}$, when inverted is $\frac{9}{8}$.

Multiply the numerators together and the denominators together; thus, $\frac{4 \times 9}{5 \times 8} = \frac{36}{40}$. This fraction, $\frac{36}{40}$, reduced to lowest terms by dividing both numerator and denominator by the same number, 4, equals $\frac{36 \div 4}{40 \div 4} = \frac{9}{10}$.

Example:

$\frac{4}{5} \div \frac{3}{4}$.
Solution:

Invert the divisor and multiply; thus, \[ \frac{4}{5} \div \frac{3}{4} = \frac{4}{5} \times \frac{4}{3} = \frac{16}{15} = 1 \frac{1}{15}. \]

**To divide a fraction by a whole number.**

Rule 15:—Invert the whole number and proceed as in the multiplication of fractions. (See Rule 10.)

Example:

\[ \frac{8}{15} \div 2. \]

Solution:

\[ \frac{2}{15} \text{ inverted} = \frac{1}{2}. \]

\[ \frac{8}{15} \div 2 = \frac{8}{15} \times \frac{1}{2} = \frac{8}{30} = \frac{4}{15}. \]

Example:

A bar of copper \( \frac{8}{12} \) of a foot long is to be cut into 3 equal parts. What will be the length of each part?

Solution:

\[ \frac{8}{12} \text{ ft.} \div 3 = \frac{8}{12} \text{ ft.} \times \frac{1}{3} = \frac{8}{36} = \frac{2}{9} \text{ ft.}, \] the length of each part.

**To divide a whole number by a fraction.**

Rule 16:—Invert the fraction and proceed as in the multiplication of a fraction by a whole number. (See Rule 11.)

Example:

\[ 46 \div \frac{4}{7}. \]

Solution:

\[ \frac{4}{7} \text{ inverted} = \frac{7}{4}. \]

\[ 46 \div \frac{4}{7} = 46 \times \frac{7}{4} = \frac{322}{4} = 80 \frac{2}{4} = 80 \frac{1}{2}. \]

Example:

A 10-pound box of rivets was divided into \( \frac{1}{2} \) pound lots. How many portions were there?

Solution:

\[ 10 \div \frac{1}{2} = 10 \times \frac{2}{1} = 20, \] the number of portions.
To divide a mixed number by a whole number.

Rule 17:—Reduce the mixed number to an improper fraction and proceed the same as to divide a fraction by a whole number. (See Rule 15.)

Example:

\[ 20 \frac{3}{5} \div 4. \]

Solution:

\[ 20 \frac{3}{5} = \frac{103}{5}. \]

\[ \frac{103}{5} \div 4 = \frac{103}{5} \times \frac{1}{4} = \frac{103}{20} = \frac{5}{2} \cdot \]

Example:

A flywheel makes \( 15 \frac{5}{6} \) revolutions in 5 seconds. How many revolutions does it make in one second?

Solution:

\[ 15 \frac{5}{6} = \frac{95}{6}. \]

\[ \frac{95}{6} \div 5 = \frac{95}{6} \times \frac{1}{5} = \frac{95}{30} = \frac{3.5}{30} = \frac{1}{6}, \text{ the number of revolutions per second.} \]

To divide a whole number by a mixed number.

Rule 18:—Reduce the mixed number to an improper fraction and proceed the same as to divide a whole number by a fraction. (See Rule 16.)

Example:

\[ 24 + 3 \frac{1}{8}. \]

Solution:

\[ 3 \frac{1}{8} = \frac{25}{8}. \]

\[ 24 + 3 \frac{1}{8} = 24 \div \frac{25}{8} = 24 \times \frac{8}{25} = \frac{192}{25} = 7 \frac{17}{25}. \]

Example:

A barrel of oil weighs 390 pounds. If a gallon weighs \( 7 \frac{1}{2} \) pounds, how many gallons are there in the barrel?
Solution:
\[ \frac{11}{2} = \frac{15}{2} \cdot \]
\[ 390 + \frac{1}{2} = 390 + \frac{15}{2} = 390 \times \frac{2}{15} = \frac{780}{15} = 52, \text{ the number of gallons.} \]

To divide a mixed number by a mixed number.

Rule 19:—Reduce both mixed numbers to improper fractions and proceed the same as to divide a fraction by a fraction. (See Rule 14.)

Example:
\[ 21 \frac{3}{7} + 12 \frac{8}{21}. \]

Solution:
\[ 21 \frac{3}{7} = \frac{150}{7}, \text{ and } 12 \frac{8}{21} = \frac{260}{21}. \]
\[ 21 \frac{3}{7} \div 12 \frac{8}{21} = \frac{150}{7} \div \frac{260}{21} = \frac{150}{7} \times \frac{21}{260} = \frac{3150}{1820} = \frac{1330}{1820} = \frac{19}{26}. \]
A shorter way to solve such problems is by cancellation.

Example:
\[ \frac{15}{165} \times \frac{21}{26} = \frac{45}{26} = \frac{19}{26}. \]

A water tank holds \( 82 \frac{1}{2} \) gallons. How many times can a pail holding \( 2 \frac{3}{4} \) gallons be filled from it?

Solution:
\[ 82 \frac{1}{2} = \frac{165}{2}, \text{ and } 2 \frac{3}{4} = \frac{11}{4}. \]
\[ 82 \frac{1}{2} \div 2 \frac{3}{4} = \frac{165}{2} \div \frac{11}{4} = \frac{165}{2} \times \frac{4}{11}. \]
By cancellation:
\[ \frac{15}{2} \]
\[ \frac{165}{2} \times \frac{\frac{4}{11}}{11} = 30, \text{ the number of times.} \]
PROBLEMS.

Note:—These problems should be answered one or more complete lessons at a time.

FIRST LESSON.

1. Reduce (a) \(\frac{3}{4}, \frac{2}{3}, \frac{5}{6}\) to twelfths. (b) \(\frac{4}{5}, \frac{5}{6}, \frac{7}{10}\) to thirtieths.

2. Reduce the following fractions to lowest terms: (a) \(\frac{14}{21}\); (b) \(\frac{18}{30}\);
   (c) \(\frac{72}{288}\); (d) \(\frac{288}{864}\).

3. Reduce to improper fractions: (a) \(\frac{4}{5}\); (b) \(\frac{12}{3}\); (c) \(\frac{17}{13}\);
   (d) \(\frac{49}{64}\).

4. Reduce to whole or mixed numbers: (a) \(\frac{25}{3}\); (b) \(\frac{1728}{12}\); (c) \(\frac{328}{49}\);
   (d) \(\frac{1421}{125}\).

5. Reduce to the least common denominator: (a) \(\frac{1}{4}, \frac{11}{12}, \frac{13}{18}, \frac{20}{36}\);
   (b) \(\frac{6}{9}, \frac{12}{14}, \frac{15}{18}, \frac{21}{36}\).

SECOND LESSON.

6. Find the sum of: (a) \(\frac{2}{3}, \frac{3}{5}, \frac{6}{7}\); (b) \(\frac{12}{13}, \frac{3}{4}, \frac{7}{8}\).

7. Add the fractions: (a) \(\frac{4}{5}, \frac{5}{6}, \frac{1}{15}\); (b) \(\frac{11}{12}, \frac{1}{6}, \frac{3}{4}\); (c) \(\frac{4}{13} + \frac{16}{17} + \frac{3}{125}\).

8. Find the result of: (a) \(3\frac{1}{2} + 4\frac{3}{4} + 1\frac{1}{2}\); (b) \(7\frac{2}{3} + 6\frac{1}{5} + 3\frac{4}{5}\);
   (c) \(13\frac{5}{8} + 29\frac{3}{15} + 8\frac{7}{60}\).

9. Solve: (a) \(6\frac{3}{4} + 4\frac{4}{5} + 7\frac{10}{12}\); (b) \(12\frac{3}{4} + 16\frac{1}{2} + 2\frac{3}{4}\); (c) \(7\frac{3}{8} + 19\frac{4}{5} + 14\frac{4}{12}\).
10. Four coils of wire weighed $45 \frac{1}{2}$ pounds, $47 \frac{3}{4}$ pounds, $38 \frac{7}{8}$ pounds, and $39 \frac{1}{2}$ pounds, respectively. What was the total weight of the wire?

11. What is the total length of 3 pieces of cable $25 \frac{3}{8}$ feet, $43 \frac{1}{16}$ feet, and $29 \frac{3}{4}$ feet long, respectively?

THIRD LESSON.

12. Subtract: (a) $\frac{3}{4}$ from $\frac{7}{9}$; (b) $\frac{13}{15}$ from $\frac{17}{19}$.

13. Subtract: (a) $\frac{3}{5}$ from $\frac{9}{10}$; (b) $\frac{7}{21}$ from $\frac{3}{7}$; (c) $\frac{13}{15}$ from $\frac{27}{25}$.

14. Solve: (a) $28 - \frac{1}{3}$; (b) $63 - \frac{2}{3}$; (c) $23 \frac{1}{7} - 15$.

15. Solve: (a) $12 \frac{1}{6} - \frac{8}{6}$; (b) $59 \frac{4}{12} - 40 \frac{7}{9}$.

16. In a pump, the diameter of the steam cylinder is $18 \frac{1}{2}$ inches and the diameter of the water cylinder is $10 \frac{1}{4}$ inches. How much larger in diameter is the steam cylinder than the water cylinder?

17. Two pieces of iron, $5 \frac{3}{4}$ inches and $6 \frac{7}{8}$ inches long, respectively, are cut from a rod $24 \frac{3}{8}$ inches long. How long is the remaining piece?

FOURTH LESSON.

18. Find the product of: (a) $\frac{4}{5} \times \frac{6}{7}$; (b) $12 \times \frac{5}{6}$.

19. Multiply: (a) $\frac{6}{7} \times \frac{8}{9} \times \frac{3}{4}$; (b) $9 \times \frac{16}{27} \times \frac{35}{45}$.

20. Multiply: (a) $7 \times \frac{3}{5}$; (b) $15 \times \frac{4\frac{1}{2}}{11}$; (c) $21 \times \frac{13}{11}$. 
21. Find the product of: (a) \(12\frac{1}{2} \times 8\frac{1}{3}\); (b) \(13\frac{1}{5} \times 7\).

22. What is the weight of \(3\frac{2}{3}\) miles of copper wire if it weighs \(6\frac{1}{4}\) pounds per 100 feet? There are 5280 feet in a mile.

23. What will be the cost of \(85\frac{1}{2}\) feet of copper wire at \(11\frac{1}{4}\) cents per foot?

**Fifth Lesson.**

24. Divide: (a) \(\frac{5}{6}\) by \(\frac{2}{3}\); (b) \(\frac{84}{99}\) by \(\frac{48}{77}\).

25. Divide: (a) \(\frac{18}{25}\) by 36; (b) \(26\frac{2}{3}\) by 3.

26. Solve: (a) \(10 + \frac{2}{5}\); (b) \(100 + 2\frac{6}{7}\); (c) \(17\frac{1}{9} + 13\).

27. Solve: (a) \(21\frac{3}{7} + 12\frac{8}{21}\); (b) \(\frac{45}{80} + \frac{90}{112}\) \(\div\) \(\frac{27}{40} \times \frac{112}{135}\).

28. If the circumference, or distance around the rim, of a locomotive driving wheel is \(21\frac{2}{3}\) feet, how many revolutions will the wheel make in traveling 784 feet?

29. In a telegraph office there are 12 relays having the same kind of windings. The sum of their resistances is \(480\frac{2}{3}\) ohms. What is the resistance of each?