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ARITHMETIC

DEFINITIONS

**Arithmetic** is the science of numbers and the art of using them.

A **unit** is one, or a single thing; for example, *one, one* track, *one* engine, *one* signal.

A **digit** is one of the figures by which numbers are expressed. The digits are 1, 2, 3, 4, 5, 6, 7, 8, 9.

*Note.*—Most authorities do not consider 0 as a digit.

A **number** is a unit or a collection of units; for example, *one* engine, *three* tracks, *five* cars, 45. Numbers may be expressed by words or characters and may be either **concrete** or **abstract**.

An **integer** or integral number is a whole number; for example, 3, 5, 10, 13.

A **concrete number** is a number that is applied to some particular kind of object or unit; for example, *four poles, five switches, seven lamps*.

An **abstract number** is a number that is not applied to some particular kind of object or unit; for example, *two, three, five, ten, six, three*.

**Like numbers** are numbers that consist of the same kind of units; for example, *six tracks, four tracks*.

**Unlike numbers** are numbers that do not consist of the same kind of units; for example, *three signals and five days, six cars and eight wheels*.

The fundamental operations in arithmetic are notation, numeration, addition, subtraction, multiplication and division.

**NOTATION AND NUMERATION**

**Notation** is the method of representing numbers by figures or letters.

**Numeration** is the method of reading numbers which have been expressed by figures or letters.

The **Arabic System** of notation is a method of expressing numbers by figures.

This method uses ten different figures to represent numbers:

**Figures** 0 1 2 3 4 5 6 7 8 9

**Names** naught one two three four five six seven eight nine

The first character, 0, has no value when standing alone, and is called **naught, cipher or zero**.
A number expressed by a single figure, as 5, represents the number of units it contains, which in this case is five.

A number expressed by two figures, as 35, means that the first figure, counting from the right, represents the number of units, which in this case is 5; the second figure, counting from the right, represents the number of tens, which in this case is 3.

It then follows that a number containing three figures, as 135, means that the first figure, counting from the right, represents the number of units; the second the number of tens; the third the number of hundreds.

In reading a number it is customary to point it off into periods of three figures each, commencing with the right-hand figure, or units. A comma (,) is used to separate these periods.

<table>
<thead>
<tr>
<th>Trillions</th>
<th>Hundred billions</th>
<th>Ten billions</th>
<th>Billions</th>
<th>Hundred millions</th>
<th>Millions</th>
<th>Hundred thousands</th>
<th>Ten thousands</th>
<th>Thousands</th>
<th>Hundreds</th>
<th>Ten</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
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</tr>
</tbody>
</table>

Example:
Read 473,562,135.

Solution:
In pointing off this number, commence at the first figure on the right and count toward the left; thus, units, tens, hundreds; insert a comma; thousands, ten thousands, hundred thousands; insert a comma; millions, ten millions, hundred millions.

The entire number will be read: Four hundred seventy-three million, five hundred sixty-two thousand, one hundred thirty-five.

The Roman System of notation is a method of expressing numbers by letters, and uses seven letters of the Roman alphabet. Thus: I represents one; V, five; X, ten; L, fifty; C, one hundred; D, five hundred; M, one thousand.

<table>
<thead>
<tr>
<th>Letters</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>1</td>
</tr>
<tr>
<td>V</td>
<td>5</td>
</tr>
<tr>
<td>X</td>
<td>10</td>
</tr>
<tr>
<td>L</td>
<td>50</td>
</tr>
<tr>
<td>C</td>
<td>100</td>
</tr>
<tr>
<td>D</td>
<td>500</td>
</tr>
<tr>
<td>M</td>
<td>1000</td>
</tr>
</tbody>
</table>

To express other numbers these letters are combined according to the following principles:
1.—Every time a letter is repeated its value is repeated. Thus: III represents 3; XX, 20; CC, 200.
2.—When a letter is placed after one of a greater value, the sum of their values is the number represented. Thus: \( \text{XV} \) represents 15; \( \text{IX}, \ 60; \text{DC}, \ 600. \)

3.—When a letter is placed before one of greater value, the difference of their values is the number represented. Thus: \( \text{IX} \) represents 9; \( \text{XL}, \ 40; \text{XC}, \ 90. \)

4.—A dash or bar placed over a letter increases its value a thousand times. Thus: \( \text{V} \) represents 5000; \( \text{X}, \ 10,000; \text{IX}, \ 9000. \)

**Roman Table**

<table>
<thead>
<tr>
<th>Letter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>one</td>
</tr>
<tr>
<td>II</td>
<td>two</td>
</tr>
<tr>
<td>III</td>
<td>three</td>
</tr>
<tr>
<td>IV</td>
<td>four</td>
</tr>
<tr>
<td>V</td>
<td>five</td>
</tr>
<tr>
<td>VI</td>
<td>six</td>
</tr>
<tr>
<td>VII</td>
<td>seven</td>
</tr>
<tr>
<td>VIII</td>
<td>eight</td>
</tr>
<tr>
<td>IX</td>
<td>nine</td>
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<tr>
<td>X</td>
<td>ten</td>
</tr>
<tr>
<td>XI</td>
<td>eleven</td>
</tr>
<tr>
<td>XIV</td>
<td>fourteen</td>
</tr>
<tr>
<td>XV</td>
<td>fifteen</td>
</tr>
<tr>
<td>XIX</td>
<td>nineteen</td>
</tr>
<tr>
<td>XX</td>
<td>twenty</td>
</tr>
<tr>
<td>XXX</td>
<td>thirty</td>
</tr>
<tr>
<td>XL</td>
<td>forty</td>
</tr>
<tr>
<td>L</td>
<td>fifty</td>
</tr>
<tr>
<td>LX</td>
<td>sixty</td>
</tr>
<tr>
<td>LXX</td>
<td>seventy</td>
</tr>
<tr>
<td>LXXX</td>
<td>three hundred</td>
</tr>
<tr>
<td>CC</td>
<td>two hundred</td>
</tr>
<tr>
<td>DC</td>
<td>six hundred</td>
</tr>
<tr>
<td>CM</td>
<td>nine hundred</td>
</tr>
<tr>
<td>M</td>
<td>one thousand</td>
</tr>
<tr>
<td>MM</td>
<td>two thousand</td>
</tr>
<tr>
<td>MCXXX</td>
<td>one thousand one hundred twenty</td>
</tr>
<tr>
<td>MCMXIII</td>
<td>one thousand nine hundred thirteen</td>
</tr>
</tbody>
</table>

Example:

Express 76 in Roman notation.

Solution:

\[ L = 50; \quad XX = 10 + 10 = 20; \quad VI = 5 + 1 = 6. \]

Then 76 equals \( 50 + 20 + 6 = \text{LXXVI}. \)

Example:

Express 725 in Roman notation.

Solution:

\[ D = 500; \quad CC = 100 + 100 = 200; \quad XX = 10 + 10 = 20; \quad V = 5. \]

Then 725 equals \( 500 + 200 + 20 + 5 = \text{DCCXXV}. \)
Example:
Read MCLV.

Solution:
M = 1000, C = 100, L = 50, V = 5.
Then MCLV would be read one thousand one hundred fifty-five.

ADDITION

Addition is the process of finding the sum of two or more numbers. The sign of addition is +, and is read plus. It means that the numbers between which it is placed are to be added. For example, 6 + 7 is read, six plus seven, and indicates that 7 is to be added to 6.

The sign of equality is =, and is read equals, or equal to. For example, 8 + 6 = 14, is read, 8 plus 6 equals 14, and indicates that the sum of 8 and 6 is 14.

Like numbers can be added, but unlike numbers cannot. For example, six cars can be added to ten cars, but six cars cannot be added to eight rails.

Rule 1: To add numbers, place them directly under each other, being careful to place units under units, tens under tens, hundreds under hundreds, thousands under thousands, etc., and draw a line beneath.

Commence at the right, add each column separately, and write the sum underneath, if it is less than 10.

Example:
What is the sum of 13 + 134 + 241 + 1?

Solution:

<table>
<thead>
<tr>
<th>Hundreds</th>
<th>Tens</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
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</tr>
</tbody>
</table>

Place the numbers so that figures of the same order are in the same column, draw a line under them and commence at the right to add. Add mentally; thus, 1 plus 1 is 2, plus 4 is 6, plus 3 is 9, which is the sum of the figures in the units column. Place the 9 directly beneath this column as the first, or units, figure in the sum. The sum of the figures in the next, or tens, column is 4 plus 3 is 7, plus 1 is 8. Place the 8 directly beneath as the second, or tens, figure in the sum. 2 plus 1 is 3. This is the sum of the figures in the next, or hundreds, column. Place the 3 directly beneath as the third, or hundreds, figure in the sum. The sum is then 389.
Rule 2:—If the sum of any column is equal to or greater than 10, place the right-hand figure under that column and add the remaining figures to the next column. Write the entire sum of the last column.

Example:

What is the sum of $3213 + 45 + 365 + 4 + 85$?

Solution:

The sum of the figures in the first, or units, column is 22 units, or 2 tens and 2 units. Place the 2 units as the first, or right-hand, figure of the sum and add the 2 tens to the figures in the tens column. The sum of the figures in the tens column is 19, and when added to the 2 tens carried from the units column, is 21 tens, which equals 2 hundreds and 1 ten. Place the 1 as the second figure of the sum, and add the 2 hundreds to the figures in the next, or hundreds, column. The sum of the figures in the hundreds column is 5 hundreds, which is added to the 2 hundreds, carried over from the tens column, making 7 hundreds. Write the 7 as the third, or hundreds, figure in the sum. There is nothing to carry to the next column because 7 is less than 10. The sum of the figures in the next column is 3 thousands, which will be the fourth figure in the sum. The sum is then 3712.

Proof:—Begin at the top and add each column downward. If the same result is then obtained as when adding upward, the sum is probably correct.

**SUBTRACTION**

Subtraction is the process of finding the difference between two numbers.

The minuend is the larger number from which a smaller number is to be subtracted.

The subtrahend is the smaller number which is to be subtracted.

The remainder is the difference between the numbers.

The sign of subtraction is $-$, and is read minus. Thus, $8 - 5$ means that 5 is to be taken from 8, and is read 8 minus 5.
Rule 3:—Place the smaller number, or subtrahend, under the larger number, or minuend, being careful to place units under units, tens under tens, etc., as in addition, and draw a line beneath.

Commence at the right-hand, or units, column; subtract each figure of the subtrahend from the figure above it in the minuend, and write the remainder beneath.

Example:

What is the difference between 365 and 124, which may also be written 365—124.

Solution:

\[
\begin{array}{c}
365 \\
124 \\
241
\end{array}
\]

Rule 4:—If any figure of the subtrahend is greater than the figure above it in the minuend, add 10 to the latter and then subtract. Subtract 1 from the next term to the left in the minuend and proceed as before.

Example:

What is the difference between 763 and 479.

Solution:

\[
\begin{array}{c}
763 \\
479 \\
284
\end{array}
\]
Example:

Find the difference between 6502 and 521.

Solution:

\[
\begin{array}{c|c|c|c|c}
\text{Thousands} & \text{Hundreds} & \text{Tens} & \text{Units} \\
\hline
6 & 5 & 0 & 2 \\
5 & 2 & 1 & \text{Minuend} \\
5 & 9 & 8 & 1 & \text{Subtrahend} \\
\end{array}
\]

1 from 2 equals 1 unit, the right-hand or units figure in the remainder. 2 tens cannot be subtracted from 0 tens; so 1 is borrowed from the hundreds column of the minuend and added to the figure in the tens column. 1 hundred equals 10 tens, which, added to the 0 tens makes 10 tens. 2 from 10 equals 8, the second, or tens, figure in the remainder. Since 1 hundred was taken from 5 hundreds, there remain only 4 hundreds in the minuend. 5 hundreds cannot be subtracted from 4 hundreds, so 1 is borrowed from the thousands column in the minuend and added to the figure in the hundreds column. 1 thousand equals 10 hundreds which, added to the 4 hundreds, equals 14 hundreds. 5 subtracted from 14 leaves 9, the third, or hundreds, figure in the remainder. 1 was taken from the thousands column in the minuend, leaving 5. Since there is no figure in the thousands column in the subtrahend to be subtracted the fourth, or thousands, figure in the remainder is 5. The remainder or difference is then 5981.

Proof:—Add the subtrahend and remainder together, and their sum will equal the minuend, if the subtraction is correct.

**MULTIPLICATION**

Multiplication is the process of taking one number as many times as there are units in another.

The multiplicand is the number to be multiplied.

The multiplier is the number showing how many times the multiplicand is to be taken.

The product is the result obtained by multiplying one number by another.

The sign of multiplication is \( \times \), and is read multiplied by, or times for example, \( 8 \times 5 \) is read 8 multiplied by 5, or 8 times 5.
### Multiplication Tables

<table>
<thead>
<tr>
<th>1 × 1 = 1</th>
<th>2 × 1 = 2</th>
<th>3 × 1 = 3</th>
<th>4 × 1 = 4</th>
<th>5 × 1 = 5</th>
<th>6 × 1 = 6</th>
<th>7 × 1 = 7</th>
<th>8 × 1 = 8</th>
<th>9 × 1 = 9</th>
<th>10 × 1 = 10</th>
<th>11 × 1 = 11</th>
<th>12 × 1 = 12</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 × 2 = 2</td>
<td>2 × 2 = 4</td>
<td>3 × 2 = 6</td>
<td>4 × 2 = 8</td>
<td>5 × 2 = 10</td>
<td>6 × 2 = 12</td>
<td>7 × 2 = 14</td>
<td>8 × 2 = 16</td>
<td>9 × 2 = 18</td>
<td>10 × 2 = 20</td>
<td>11 × 2 = 22</td>
<td>12 × 2 = 24</td>
</tr>
<tr>
<td>1 × 3 = 3</td>
<td>2 × 3 = 6</td>
<td>3 × 3 = 9</td>
<td>4 × 3 = 12</td>
<td>5 × 3 = 15</td>
<td>6 × 3 = 18</td>
<td>7 × 3 = 21</td>
<td>8 × 3 = 24</td>
<td>9 × 3 = 27</td>
<td>10 × 3 = 30</td>
<td>11 × 3 = 33</td>
<td>12 × 3 = 36</td>
</tr>
<tr>
<td>1 × 4 = 4</td>
<td>2 × 4 = 8</td>
<td>3 × 4 = 12</td>
<td>4 × 4 = 16</td>
<td>5 × 4 = 20</td>
<td>6 × 4 = 24</td>
<td>7 × 4 = 28</td>
<td>8 × 4 = 32</td>
<td>9 × 4 = 36</td>
<td>10 × 4 = 40</td>
<td>11 × 4 = 44</td>
<td>12 × 4 = 48</td>
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<td>2 × 5 = 10</td>
<td>3 × 5 = 15</td>
<td>4 × 5 = 20</td>
<td>5 × 5 = 25</td>
<td>6 × 5 = 30</td>
<td>7 × 5 = 35</td>
<td>8 × 5 = 40</td>
<td>9 × 5 = 45</td>
<td>10 × 5 = 50</td>
<td>11 × 5 = 55</td>
<td>12 × 5 = 60</td>
</tr>
<tr>
<td>1 × 6 = 6</td>
<td>2 × 6 = 12</td>
<td>3 × 6 = 18</td>
<td>4 × 6 = 24</td>
<td>5 × 6 = 30</td>
<td>6 × 6 = 36</td>
<td>7 × 6 = 42</td>
<td>8 × 6 = 48</td>
<td>9 × 6 = 54</td>
<td>10 × 6 = 60</td>
<td>11 × 6 = 66</td>
<td>12 × 6 = 72</td>
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<tr>
<td>1 × 7 = 7</td>
<td>2 × 7 = 14</td>
<td>3 × 7 = 21</td>
<td>4 × 7 = 28</td>
<td>5 × 7 = 35</td>
<td>6 × 7 = 42</td>
<td>7 × 7 = 49</td>
<td>8 × 7 = 56</td>
<td>9 × 7 = 63</td>
<td>10 × 7 = 70</td>
<td>11 × 7 = 77</td>
<td>12 × 7 = 84</td>
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<td>2 × 8 = 16</td>
<td>3 × 8 = 24</td>
<td>4 × 8 = 32</td>
<td>5 × 8 = 40</td>
<td>6 × 8 = 48</td>
<td>7 × 8 = 56</td>
<td>8 × 8 = 64</td>
<td>9 × 8 = 72</td>
<td>10 × 8 = 80</td>
<td>11 × 8 = 88</td>
<td>12 × 8 = 96</td>
</tr>
<tr>
<td>1 × 9 = 9</td>
<td>2 × 9 = 18</td>
<td>3 × 9 = 27</td>
<td>4 × 9 = 36</td>
<td>5 × 9 = 45</td>
<td>6 × 9 = 54</td>
<td>7 × 9 = 63</td>
<td>8 × 9 = 72</td>
<td>9 × 9 = 81</td>
<td>10 × 9 = 90</td>
<td>11 × 9 = 99</td>
<td>12 × 9 = 108</td>
</tr>
<tr>
<td>1 × 10 = 10</td>
<td>10 × 10 = 100</td>
<td>11 × 10 = 110</td>
<td>12 × 10 = 120</td>
<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1 × 11 = 11</td>
<td>10 × 11 = 110</td>
<td>11 × 11 = 121</td>
<td>12 × 11 = 132</td>
<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1 × 12 = 12</td>
<td>10 × 12 = 120</td>
<td>11 × 12 = 132</td>
<td>12 × 12 = 144</td>
<td></td>
<td></td>
<td></td>
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<td></td>
</tr>
</tbody>
</table>

**Note.**—These tables should be carefully studied and memorized.
To multiply a number when the multiplier consists of one figure only.

Rule 5:—Write the figure in the multiplier directly under the first figure on the right, or units place, in the multiplicand. Begin at the right and multiply each figure in the multiplicand by the multiplier.

Example:
Multiply 762 by 7.

Solution:

\[
\begin{array}{c|c|c|c}
& \text{Hundreds} & \text{Tens} & \text{Units} \\
\hline
\text{Multiplicand} & 7 & 6 & 2 \\
\text{Multiplier} & & & 7 \\
\hline
\text{Product} & 5 & 3 & 3 \ 4
\end{array}
\]

Write the multiplier under the multiplicand and draw a line beneath. Beginning at the right to multiply, 7 times 2 units are 14 units or 1 ten and 4 units. Write the 4 in the units place in the product, and reserve the tens to add to the next product. 7 times 6 tens are 42 tens, which, added to the 1 ten, equals 43 tens, or 4 hundreds plus 3 tens. Write the 3 in the tens place in the product and reserve the 4 hundreds to add to the next product. 7 times 7 hundreds are 49 hundreds, which, added to the 4 hundreds, equals 53 hundreds, or 5 thousands and 3 hundreds. The figures 5 and 3 are written in their proper places as shown in the above solution. The product is then 5334.

To multiply a number when the multiplier consists of two or more figures.

Rule 6:—Write the multiplier under the multiplicand placing units under units, tens under tens, hundreds under hundreds, thousands under thousands, etc., and draw a line beneath.

Begin at the right and multiply each figure of the multiplicand by the multiplier, writing the right-hand figure of each partial product so obtained directly under the figure in the multiplier which produced it.

Add the partial products together, and their sum will be the entire product.

Example:
Multiply 423 by 656.

Solution:

\[
\begin{array}{c|c|c|c|c|c}
& \text{Hundreds} & \text{Tens} & \text{Units} \\
\hline
\text{Multiplicand} & 4 & 2 & 3 \\
\text{Multiplier} & 6 & 5 & 6 \\
\hline
\text{First partial product} & 2 & 5 & 3 & 8 \\
\text{Second partial product} & 2 & 1 & 1 & 5 \\
\text{Third partial product} & 2 & 5 & 3 & 8 \\
\hline
\text{Product} & 2 & 7 & 7 & 4 & 8 \ 8
\end{array}
\]

\[6 \times 423 = 2538; \ 5 \times 423 = 2115; \ 6 \times 423 = 2538.\]

The right-hand figure, 8, in the first partial product, 2538, is written directly under 6, the first figure in the multiplier, or the number multiplied by. The right-hand figure, 5, in the second partial product, 2115,
is written directly under 5, the second figure in the multiplier. The	right-hand figure, 8, in the third partial product, 2538, is written
directly under 6, the third figure in the multiplier. The sum of these
three partial products is 277488, or the product of 423 \times 656.

*When there is a cipher in the multiplier, multiply by it as well as by the other figures.*

Note.—0 times a number equals 0. Thus: \(0 \times 2 = 0\); \(0 \times 18 = 0\);
\(0 \times 154 = 0\).

**Example:**

Multiply 258 by 203.

**Solution:**

\[
\begin{array}{c}
258 \\
203 \\
\hline
774 \\
000 \\
516 \\
\hline
52374
\end{array}
\]

**Example:**

Multiply 636 by 403.

**Solution:**

\[
\begin{array}{c}
636 \\
403 \\
\hline
1908 \\
25440 \\
\hline
256308
\end{array}
\]

The work may be shortened when multiplying by a number containing a 0 by writ-
ing the first cipher of the partial product, then multiplying by the next figure of the
multiplier and writing the partial product to the left of the cipher.

**Example:**

Multiply 2432 by 2004.

**Solution:**

\[
\begin{array}{c}
2432 \\
2004 \\
\hline
9728 \\
486400 \\
4873728
\end{array}
\]

The **Significant Figures** in a whole number or decimal are those which remain after canceling all the ciphers at the right or left. Thus, the
significant figures in 12400, 0.0124, or 1240, are 124.
Annexing one cipher to the right of a number multiplies it by 10; annexing two ciphers multiplies it by 100; annexing three ciphers multiplies it by 1000, etc.

*When there are ciphers at the right of either the multiplicand or multiplier, or both, take the product of the numbers denoted by the significant figures, and place to the right of the product as many ciphers as are found at the right of both multiplicand and multiplier.*

*Note.*—The figures are usually set down with the significant figures under each other, the same as though no ciphers were present.

**Example:**

\[352 \times 140.\]

**Solution:**

\[
\begin{array}{c}
352 \\
140 \\
\hline
1408 \\
352 \\
\hline
49280
\end{array}
\]

*Example:*

\[1260 \times 340.\]

**Solution:**

\[
\begin{array}{c}
1260 \\
340 \\
\hline
504 \\
378 \\
\hline
428400
\end{array}
\]

There is one cipher at the right of each of the multiplicand and multiplier; so two ciphers are placed to the right of the significant figures in the product.

**Example:**

Multiply \(2600 \times 180.\)

**Solution:**

\[
\begin{array}{c}
2600 \\
180 \\
\hline
208 \\
26 \\
\hline
468000
\end{array}
\]

In this problem, there is one cipher in the multiplier and two ciphers in the multiplicand; therefore, place three ciphers to the right of the significant figures in the product.
14

DIVISION

Division is the process of finding out how many times one number or quantity is contained in another; it is the inverse of multiplication. The dividend is the number to be divided. The divisor is the number used to divide by. The quotient is the result obtained by dividing one number or quantity by another.

A partial dividend is that part of the dividend used in obtaining a figure of the quotient.

The sign of division is $\div$ and is read divided by; for example, $48 \div 6$ is read 48 divided by 6. Another way to write this is $\frac{48}{6}$.

There are two methods of dividing numbers; namely, short division and long division.

Short division is the method in which only the dividend, divisor and quotient are written, and the operations are performed mentally. It is usually employed when the divisor consists of only one figure.

Rule 7:—Begin at the left, divide each figure of the dividend by the divisor, and write the quotient beneath. If there is a remainder after any division, consider this as prefixed to the next figure of the dividend, and proceed as before.

Example:
Divide 735 by 5.

Solution:

\[
\begin{array}{c|c}
\text{Divisor} & \text{Dividend} \\
5) & 7 3 5 \\
\hline
1 & 4 7 \\
\end{array}
\]

Begin at the left to divide. 5 is contained in 7 once, with a remainder of 2. Write 1 as the first, or left-hand, figure of the quotient, and consider the 2 as placed before the 3 in the dividend, making 23. 5 is contained in 23 four times, with a remainder of 3. Write 4 as the second figure of the quotient, and consider the 3 as placed before the 5 in the dividend, making 35. 5 is contained in 35 seven times, with no remainder. Write 7 as the third figure in the quotient. The quotient is then 147.

Example:
Find the result of $18928 \div 7$.

Solution:

\[
\begin{array}{c|c}
\text{Divisor} & \text{Dividend} \\
7 & 1 8 9 2 8 \\
\hline
2 & 7 0 4 \\
\end{array}
\]

Begin at the left to divide. 7 is not contained in the first figure of the dividend, so the first two figures are used. 7 is contained in 18 two times.
with a remainder of 4. Write 2 as the first figure of the quotient and consider the 4 as placed before the 9 in the dividend, making 49. 7 is contained in 49 seven times, with no remainder. Write 7 as the second figure in the quotient. 7 is not contained in 2, so place a 0 as the third figure in the quotient and write the 2 before the 8 in the dividend, making 28. 7 is contained in 28 four times, with no remainder. Write 4 as the fourth figure in the quotient. The quotient is then 2704.

**Long division** is the method in which all the work is written out in detail, and is usually employed when the divisor consists of two or more figures.

**When the divisor consists of two or more figures.**

**Rule 8:**—*Draw lines on each side of the dividend, and place the divisor at the left. Find how many times the divisor is contained into the lowest number of left-hand figures of the dividend that will contain the divisor, and write this figure as the first figure of the quotient. Multiply the divisor by this figure; write the product under the partial dividend; subtract; and to the remainder bring down the next figure of the dividend. Divide as before and thus continue until all the figures of the dividend have been used.*

**Proof:**—*Multiply the quotient by the divisor and add the last remainder, if there be any. The result will be the dividend.*

Example:

Divide 2046492 by 84.

Solution:

<table>
<thead>
<tr>
<th>Divisor</th>
<th>Dividend</th>
<th>Quotient</th>
</tr>
</thead>
<tbody>
<tr>
<td>84</td>
<td>2046492</td>
<td>24363</td>
</tr>
<tr>
<td></td>
<td>168</td>
<td></td>
</tr>
<tr>
<td></td>
<td>366</td>
<td></td>
</tr>
<tr>
<td></td>
<td>336</td>
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<tr>
<td></td>
<td>304</td>
<td></td>
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<tr>
<td></td>
<td>252</td>
<td></td>
</tr>
<tr>
<td></td>
<td>529</td>
<td></td>
</tr>
<tr>
<td></td>
<td>504</td>
<td></td>
</tr>
<tr>
<td></td>
<td>252</td>
<td></td>
</tr>
<tr>
<td></td>
<td>252</td>
<td></td>
</tr>
</tbody>
</table>

84 is not contained in the first two figures of the dividend; so the first three are used. 84 is contained in 204, the first partial dividend, two times, with a remainder of 36. Write 2 as the first, or left-hand, figure in the quotient, and bring down the next figure in the dividend, 6, to the right of the remainder, 36, making 366, the second partial dividend. 84 is contained in 366 four times, with a remainder of 30. Write 4 as the second figure of
the quotient, and bring down the next figure in the dividend, 4, to the right of the remainder, 30, making 304, the third partial dividend. 84 is contained in 304 three times, with a remainder of 52. Write 3 as the third figure in the quotient, and bring down the next figure in the dividend, 9, to the right of the remainder, 52, making 529, the fourth partial dividend. 84 is contained in 529 six times, with a remainder of 25. Write 6 as the fourth figure in the quotient, and bring down the next figure in the dividend, 2, to the right of the remainder, 25, making 252, the fifth partial dividend. 84 is contained in 252 three times, with no remainder. Write 3 as the fifth figure in the quotient. The quotient is then 24363.

Example:
Divide 750145 by 62.

Solution:

\[
\begin{array}{ccc}
\text{Divisor} & \text{Dividend} & \text{Quotient} \\
62 & 750145 & 12099+ \\
62 & 62 & \\
62 & 130 & \\
62 & 124 & \\
62 & 614 & \\
62 & 558 & \\
62 & 565 & \\
62 & 558 & \\
62 & 7 & \text{Remainder}
\end{array}
\]

62 is contained in 75 once, with a remainder of 13. Write 1 as the first figure of the quotient, and bring down the next figure in the dividend, 0, to the right of the remainder, 13, making 130. 62 is contained in 130 two times, with a remainder of 6. Write 2 as the second figure of the quotient, and bring down the next figure in the dividend, 1, to the right of the remainder, 6, making 61. 62 is not contained in 61; so place a 0 in the quotient, and bring down the next figure in the dividend, 4, to the right of 61, making 614. 62 is contained in 614 nine times, with a remainder of 56. Write 9 as the fourth figure of the quotient and bring down the next figure in the dividend, 5, to the right of the remainder, 56, making 565. 62 is contained in 565 nine times, with a remainder of 7. Write 9 as the fifth figure of the quotient. The quotient is then 12099+.

Note.—The sign, +, following a quotient is used to denote a remainder.
FACTORS AND FACTORING

The factors of a number are those numbers which, when multiplied together, will produce that number. Thus: \(2 \times 3 \times 5 = 30\), therefore, 2, 3, and 5 are factors of 30.

Factoring is the process of finding the factors of a quantity.

An exact divisor of a number is an integer or whole number that will divide the number without leaving a remainder. Thus: 2, 3, 6, and 9 are exact divisors of 18.

A prime number is a number that has no factors except itself and 1. Thus: 2, 3, 5, 7, 11, 13 are prime numbers.

A composite number is a number that can be produced by multiplying together two or more numbers, each greater than a unit; as, 6, 8, 10, 15.

An even number is a number that is divisible by 2 without a remainder; as, 4, 6, 8, 10, 12.

An odd number is a number that is not divisible by 2 without a remainder; as, 3, 5, 7, 9, 11.

A prime number that is a factor of another number is a prime factor of that number. Thus: 2, 3, and 5 are the prime factors of 30.

An exponent is a small figure placed to the right of a number and a little above it, and shows how many times the number is used as a factor. Thus: in \(4^3\), the number 3 is the exponent, and shows that 4 is to be used 3 times as a factor, which is the same as writing \(4 \times 4 \times 4\).

Cancellation is a short method of division by rejecting the same factors in both dividend and divisor.

Rule 9:—Cancel the same or common factors in both the dividend and divisor. The result obtained by dividing the product of the remaining factors of the dividend by the product of the remaining factors of the divisor will be the quotient.

Canceling the same factor in both divisor and dividend does not change the quotient.

The numbers forming the dividend are placed above the line, and those forming the divisor are placed below it.
Example: Divide $45 \times 30 \times 7$ by $7 \times 25$.

Solution:

Place the dividend, $45 \times 30 \times 7$, over the divisor, $7 \times 25$. The 25 in the divisor and the 45 in the dividend are both divisible by 5, since $5 \times 9 = 45$, and $5 \times 5 = 25$. Cross off or cancel the 45 and write 9 over it, also cross off the 25 and write 5 under it as shown.

\[
\begin{array}{c}
9 \\
45 \times 30 \times 7 \\
7 \times 25 \\
5 \\
1
\end{array}
\]

5 in the divisor and 30 in the dividend are both divisible by 5, since $5 \times 1 = 5$, and $5 \times 6 = 30$. Cross off the 30 and write 6 over it, also cross off the 5 and write 1 under it as shown.

\[
\begin{array}{c}
9 \\
45 \times 30 \times 7 \\
7 \times 25 \\
1 \\
5 \\
6
\end{array}
\]

7 in the divisor and 7 in the dividend are both divisible by 7, since $7 \times 1 = 7$. Cross off the 7 in the dividend and write 1 over it, cross off the 7 in the divisor and write 1 under it as shown.

\[
\begin{array}{c}
9 \\
45 \times 30 \times 7 \\
7 \times 25 \\
1 \\
5 \\
1 \\
6 \\
1
\end{array}
\]

Since there are no numbers left in both the dividend or divisor, which are divisible by the same number except 1, it is impossible to cancel further. Multiply all remaining numbers in the dividend together and divide this product by the product of the remaining numbers in the divisor. The product of the remaining numbers in the dividend is $9 \times 6 \times 1 = 54$, and the product of the remaining numbers in the divisor is $1 \times 1 = 1$.

Therefore,

\[
\begin{array}{c}
9 \\
45 \times 30 \times 7 \\
7 \times 25 \\
1 \\
5 \\
45 \times 30 \times 7 \\
7 \times 25 \\
1 \\
6 \\
1
\end{array}
\]

Note.—It is customary to omit the 1's when canceling.

Note.—In solving problems in cancellation it is not necessary to show all the steps as above, these different operations being shown here to explain the various steps.
COMMON SYMBOLS

The parentheses ( ), brackets [ ], braces { }, and the vinculum —, called common symbols, are used to include numbers which are to be considered together. Thus: $10 \times (6-4)$, or $10 \times 6 - 4$ means that 4 is to be taken from 6 before multiplying by 10. These expressions are solved as follows:

$$10 \times (6-4) = 10 \times 2 = 20, \text{ or } 10 \times 6 - 4 = 10 \times 2 = 20.$$

When $+$ and $-$ are the only signs in an expression, the operations are performed in the order in which they occur, beginning at the left. Thus: $4 + 6 - 7 + 3 = 6$.

When $+$ occurs in an expression in connection with $+$, $-$, or both, the indicated division must be performed first.

Example:

$$7 + 10 \div 5 = ?$$

Solution:

The division is performed first; as, $10 \div 5 = 2$; then, $7 + 2 = 9$.

When $\times$ occurs in an expression in connection with $+$, $-$, or both, the indicated multiplication must be performed first.

Example:

$$5 \times 3 - 2 = ?$$

Solution:

The multiplication is performed first; as, $5 \times 3 = 15$; then, $15 - 2 = 13$.

When $\div$ and $\times$ are the only signs in an expression, or are succeeding signs in a series, the indicated multiplication and division are performed from left to right in the order in which they occur.

Example:

$$7 + 6 - 8 \div 2 \times 3 = ?$$

Solution:

$8 \div 2 = 4$; $4 \times 3 = 12$; $7 + 6 = 13$; $13 - 12 = 1$. 
20

PROBLEMS

Note.—These problems should be answered one or more complete lessons at a time.

FIRST LESSON

1.—Write out in figures by the Arabic system the following:
(a) Seven thousand two hundred fifty-five; (b) One million seventy-five thousand one hundred four.

2.—Write out in words the following: (a) 1705; (b) 5673.

3.—How many hundreds in (a) 2 thousands? (b) 5 thousands? (c) 50 tens?

4.—Express the following numbers by the Roman system: (a) Twenty-seven; (b) Sixty-eight; (c) Three hundred forty-five; (d) One thousand six hundred forty-five; (e) Five thousand six hundred forty-three; (f) Forty-five thousand four hundred thirty-two.

5.—Write out in words the following numbers: (a) CDXII; (b) MDCCLXXVII.

SECOND LESSON

6.—Find the sum of: (a) 734 + 241 + 123 + 390; (b) 1096 + 6824 + 1365 + 2341.

7.—Find the sum of: (a) 1635 + 26 + 18072 + 5 + 2612; (b) 10625 + 2345 + 23 + 267521 + 9871.

8.—(a) A car travels 231 miles on Monday, 176 miles on Tuesday, 192 miles on Wednesday, 201 miles on Thursday, 181 miles on Friday, 76 miles on Saturday, and 159 miles on Sunday. What was the car mileage for the week? (b) If the car in the above problem had traveled 5 miles additional each day, what would have been the car mileage for the week?

9.—The resistances of 4 telegraph relays are 435, 375, 392, and 410 ohms, respectively. What is the sum of their resistances?

10.—One man’s wages amounted to $96 for the month, 2 men receive $75 each, and 3 men receive $60 each. What is the total amount of their pay?

THIRD LESSON

11.—(a) From 7362 take 6151; (b) from 125634 take 24313.

12.—Find the result of: (a) 9137 − 8038; (b) 123065 − 70076.

13.—(a) From 1264 + 2335 + 726 subtract 3416; (b) From 622 + 1236 + 9475 + 36 subtract 8964 + 23.
14.—(a) From a coal pile containing 110,560 pounds, one engine took 7240 pounds, another 5630 pounds, and a third 6450 pounds. How many pounds of coal remained? (b) From a keg containing 725 bolts, there were taken at different times 47, 35, and 60 bolts, respectively. How many bolts remained?

15.—A train going from New York to Pittsburgh in the first hour traveled 48 miles, the second hour 42 miles, the third hour 45 miles, the fourth hour 35 miles, and the fifth hour 31 miles. How far is the train from Pittsburgh at the end of the fifth hour, the distance between New York and Pittsburgh being 440 miles?

FOURTH LESSON

16.—Multiply: (a) 12694 × 8; (b) 52762 × 7.
17.—Multiply: (a) 21456 × 45; (b) 93762 × 96.
18.—Multiply: (a) 7625 × 4398; (b) 6452 × 3761.
19.—If it requires 124 tons of steel worth $25 a ton to build one mile of railroad track, what will be the cost of the steel to build a track 235 miles long?

20.—How many miles will an engine travel in a year if it makes a trip from New York to Philadelphia (90 miles) and back every day except Sunday? There are 52 weeks in a year.

FIFTH LESSON

21.—Find the quotient of: (a) 2640 ÷ 6; (b) 21204 ÷ 9.
22.—Find the quotient of: (a) 18096 ÷ 29; (b) 224250 ÷ 6.
23.—Find the quotient of: (a) 3635152 ÷ 536; (b) 11657070 ÷ 2907.
24.—If the driving wheel on a locomotive is 18 feet in circumference, how many revolutions will it make in going from Philadelphia to Pittsburgh (350 miles)? There are 5280 feet in a mile.
25.—How many rails each 33 feet long will be necessary to lay a piece of track 24 miles long?

SIXTH LESSON

26.—Find the prime factors of: (a) 325; (b) 2310.
27.—Find the value of: (a) (860 + 980 − 1120) ÷ 45; (b) (320 − 98) × (860 − 145).
28.—Find the value of: (a) $(89 + 96 - 47) \div 6$; (b) $(3014 - 2601) \times \frac{2477 - 1325}{295 \div 5}$.

29.—Solve by cancellation: (a) $(12 \times 14 \times 16) \div (6 \times 7 \times 8)$; (b) $(20 \times 32 \times 35) \div (4 \times 5 \times 14)$.

30.—Solve by cancellation: (a) $(115 \times 110 \times 300 \times 1050) \div (23 \times 125 \times 35 \times 30)$; (b) $(625 \times 729 \times 175) \div (75 \times 81 \times 125 \times 147)$. 