THE PENNSYLVANIA RAILROAD COMPANY
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EDUCATIONAL COURSE

PAMPHLET E-5

ELEMENTARY ELECTRICITY

MAGNETIC INDUCTION

OFFICE OF
SUPERINTENDENT OF TELEGRAPH
PHILADELPHIA

PAMPHLET E-5

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MAGNETIC INDUCTION

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1. HISTORICAL.—It was not long after the discovery of the fact that there was a relation between electric current and magnetism, and that motion was produced when an electric current was brought under the influence of a magnet, that Michael Faraday and Joseph Henry conceived the idea of a reverse action through which an electric current might be produced by the motion of a wire in a magnetic field. This thought underwent development until about 1830 when Faraday brought forth the following statement: When a conductive circuit is placed in a magnetic field and the intensity of the field or the relative position of either the field or the circuit is altered, then, in general, an electric current is caused to flow in the conductive circuit by, and during the continuance of, the change. In other words, there is a tendency for an electric current to flow in a conductor when it is moved in a magnetic field so as to cut the lines of force of the field; and furthermore, any change in the magnetic field surrounding a wire tends to set up an electric current in the wire, exactly as any change in an electric current which flows in a wire causes a corresponding change in the magnetic field about the wire. In this great discovery lies the principle of the operation of electric generators, transformers, induction coils, telephones, etc.

2. EXAMPLES TO ILLUSTRATE FARADAY'S PRINCIPLE.—Probably the simplest magnetic field in which the above principle may be tried out is that of a simple bar magnet. If the wire, AB, in Fig. 1, is moved swiftly across in front of the N pole of the bar magnet, an electromotive force will be created or generated (generally spoken of as *induced*) in AB. This electromotive force—hereafter spoken of as e.m.f.—is evident when the circuit is completed through a galvanometer, G. A galvanometer is a very sensitive instrument, and is used to detect and measure small electric currents. In this case, when the wire, AB, is moved across the pole face, or end of the magnet, the pointer on the galvanometer would move, which movement indicates that a current is flowing through the instrument. Here then is illustrated the fact that if a conductor is moved through a magnetic field, an e.m.f. is generated and current flows through the conducting circuit, AB. The movement of the conductor through the magnetic field produces the same effect as a primary cell would produce by chemical action if put in place of the conductor. As soon as the wire ceases to move through the magnetic field no current flows, which result indicates the fact that there is no e.m.f. being generated. If AB is brought back across the face of the pole, current flows again, but in the opposite direction, as would be indicated by the galvanometer. It appears, therefore, that the direction of the induced e.m.f. is dependent on the direction the conductor moves in cutting the magnetic field.

Suppose we have two large magnet poles—a north pole and a south pole—facing each other as in Fig. 2a. Fig. 2b is a view looking down on

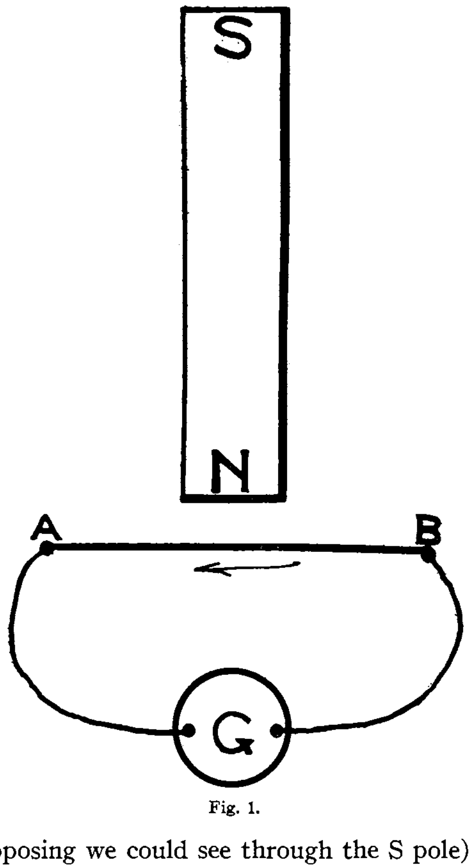

Fig. 1.

the N pole (supposing we could see through the S pole). The dots indicate the lines of force coming up from the pole toward the reader. AB is a simple straight wire. If AB is moved in the direction as indicated by the arrow so as to actually cut the imaginary lines of the field, there will be a *difference of potential* created between A and B so long as the motion continues; in other words, an e.m.f. is set up or induced in the conductor along its length, whereby one end is raised to a higher potential than the other. If AB were a small pipe filled with water, and held perfectly level, there would be no tendency for the water to flow;

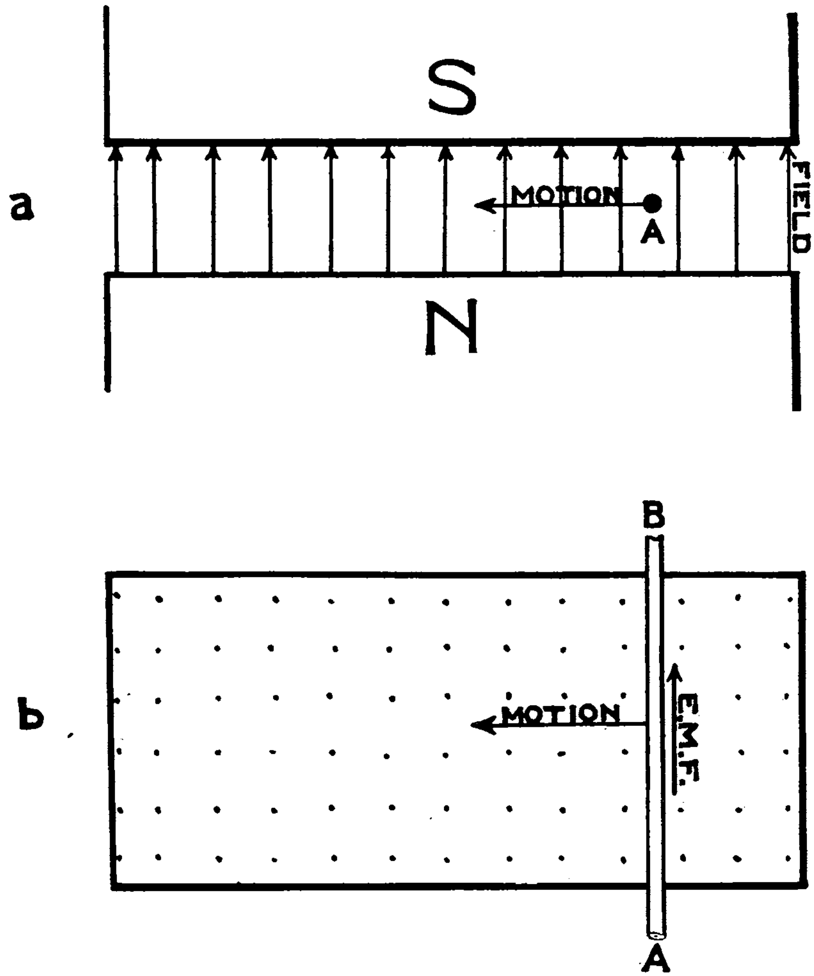

Fig. 2.

but if one end, A, is raised above the level of the other end, B, the water will tend to flow from A toward B; that is, A is raised to a higher *potential* than B. *Potential* implies power. Water on a mountain top has a greater potential—greater inherent power, or greater ability to do work—

than water in the valley. Similarly, when the wire AB is caused to cut the lines of force in the direction indicated, A is raised to a higher electrical potential than B and current tends to flow—but does not until the circuit is complete—from A to B. The wire AB—called an *inductor* from the fact that an e.m.f. is induced in it—may be regarded as a small primary cell on open circuit; no current is flowing, but the e.m.f. nevertheless exists. It is important to note that it requires no more energy to move AB across the pole face than it does to swing a piece of rope of equal weight in air. However, this statement holds true only until the two ends, A and B, are electrically joined so as to complete a closed circuit of conducting material, as in Fig. 1, when a current will flow, so long as the e.m.f. is present or as long as the wire, AB, cuts the magnetic field.

It does not matter whether the conductor is moved across the field, or whether the lines of force are moved across the conductor by movement of the magnet between whose pole pieces the field exists; or both field and conductor may be moved in opposite directions. For example, the pole faces in Fig. 2 can be moved toward the right while the conductor is held perfectly still; and exactly the same phenomena will result as when the poles are stationary and the wire moved toward the left. In all cases where the conductor cuts the lines, either by its own movement or by reason of the moving magnetic field, it becomes the source of an induced e.m.f. Faraday had been obtaining current from his primary cells by means of chemical action; now he was soon able to obtain it by means of wires continuously cutting a magnetic field, which action is called ELECTROMAGNETIC INDUCTION, because it refers to inducing current in a wire by cutting the field of an electromagnet with that wire. It might here be stated that, while Faraday's principle is utilized very familiarly every day, the ultimate reason *why* an e.m.f. is induced by the relative movement of an electrical conductor and a magnetic field is still in the realms of mystery.

3. MAGNITUDE OF E. M. F.—The amount or magnitude of e.m.f. induced in an inductor depends in a sense on but one factor; namely, the rate at which the inductor cuts the lines of force. It does not matter of what the material is composed so long as it is a conductor, since all inductors cutting a magnetic field at the same rate have the same e.m.f. induced in them. To be specific, if an inductor of any length and of any material cuts 10^8 or 100,000,000 lines of flux in one second, an e.m.f. of one volt will be induced in that inductor. It is not necessary that the circuit be closed and that current shall flow to have this e.m.f. A suspended hammer has power to deliver a blow; but it need not deliver the blow. Similarly, an e.m f. may be generated in an inductor and yet have no current flowing. Suppose the pole face of a magnet has 100,000,000 lines of force emanating from it. If an inductor cuts across this face in a second, it will have an e.m.f. of one volt induced in it during the time it moves across the pole face; if in $\frac{1}{10}$ of a second, it will have 10 volts induced, because it is moving 10 times as fast. Furthermore, if the flux is evenly distributed across the pole face, the 10 volts (in the latter case) will be induced the instant the conductor strikes the field and will continue to exist until the conductor leaves the field. The following example will make this clear:

Example 1.

A wire passes 40 times a second across the pole face of a magnet where 15,000 lines emanate from every square centimeter; that is, there is a *flux density* of 15,000 lines per square centimeter. The dimensions of the pole face are 30 cm. by 20 cm. What e.m.f. is induced in the wire?

Solution:

The total number of lines emanating from the pole equals $30 \times 20 \times$ 15,000 lines $=$ 9,000,000 lines. In one second, $40 \times$ 9,000,000 lines $=$ 360,000,000 lines are cut. Cutting 100,000,000 lines a second causes an e.m.f. of one volt. Cutting 360,000,000 lines a second causes an e.m.f. of $\frac{360000000}{100000000} = 3.6$ volts. It is not necessary to cut across this pole 40 times to create 3.6 volts; the very first time the inductor strikes the lines, it has 3.6 volts generated. That is, the induced e.m.f. depends upon the rate at which the lines of force are cut *at any instant*, and not upon how long the cutting is kept up. A similar example of this is that a train may be going at a speed of 70 miles per hour and yet may be but 3 miles from the station. Thus the above inductor may cut lines at the rate 360,000,000 lines a second and create an e.m.f. of 3.6 volts and yet actually cut but 1,000,000 or less lines. Of course the 1,000,000 lines must be cut in $\frac{1}{360}$ of a second, or at the rate of 360,000,000 lines per second. This e.m.f. which is created is not like the charge on a condenser; it lasts only while the field is being cut and disappears as soon as this action is stopped.

The rate of cutting flux, and, as a consequence the induced e.m.f., is directly proportional to the strength of the magnetic field. Suppose the wire AB (Fig. 1), in cutting the magnetic field, has two volts induced in it. If a magnet of twice the strength of the one illustrated is used, and if AB cuts the field with the same speed as in the first case, an e.m.f. of two times two volts, or four volts, will be set up. It is quite evident that, if the magnetic field is twice as strong, the rate of cutting flux will be doubled, and, as a result, the induced e.m.f., which is proportional to the rate of cutting, will be doubled.

Example 2.

A wire cuts across the face of a magnet and has 0.8 volt induced in it. What voltage would have been induced if the magnet had been 7.5 times as powerful?

Solution:

Since the induced e.m.f. is directly proportional to the strength of the magnetic field, the e.m.f. induced in the second case would have been 7.5×0.8 volt $= 6$ volts.

A second factor on which the rate of cutting depends is the length of the inductor in the magnetic field. If the inductor of Fig. 2 were one half as long, the e.m.f. induced would be but one half as great. If AB (Fig. 2) were twice as long, it is probable that but a very little increase in e.m.f. would be noticed, because the additional part would not be within the limits of the magnetic field. The induced e.m.f. is proportional only to the length of inductor within the magnetic field.

Example 3.

A wire, ten centimeters long, in cutting a magnetic field, has 1.2 volts induced in it. What e.m.f. would have been induced in the wire if it had been seventeen centimeters long, assuming that the total length of wire is in the magnetic field?

Solution:

Since the e.m.f. induced in an inductor is proportional to the length in the magnetic field, the e.m.f. induced in the second case would have been $\frac{17}{10} \times 1.2$ volts $= 2.04$ volts.

A third factor on which the induced e.m.f. depends is the speed at which the inductor moves. It is evident that the rate of cutting lines is directly proportional to the speed of the inductor. A fast-moving inductor will have a greater induced e.m.f. than a slow-moving one; for example, an inductor moving in a magnetic field at a speed of 5000 feet per minute will have twice as great an e.m.f. induced as the same inductor moving at a speed of 2500 feet per minute in the same field.

A fourth factor influencing the rate of cutting is the angle at which the inductor cuts the magnetic field. Suppose the wire, MP, in Fig. 3a is moved across the magnetic field at right angles to the lines of

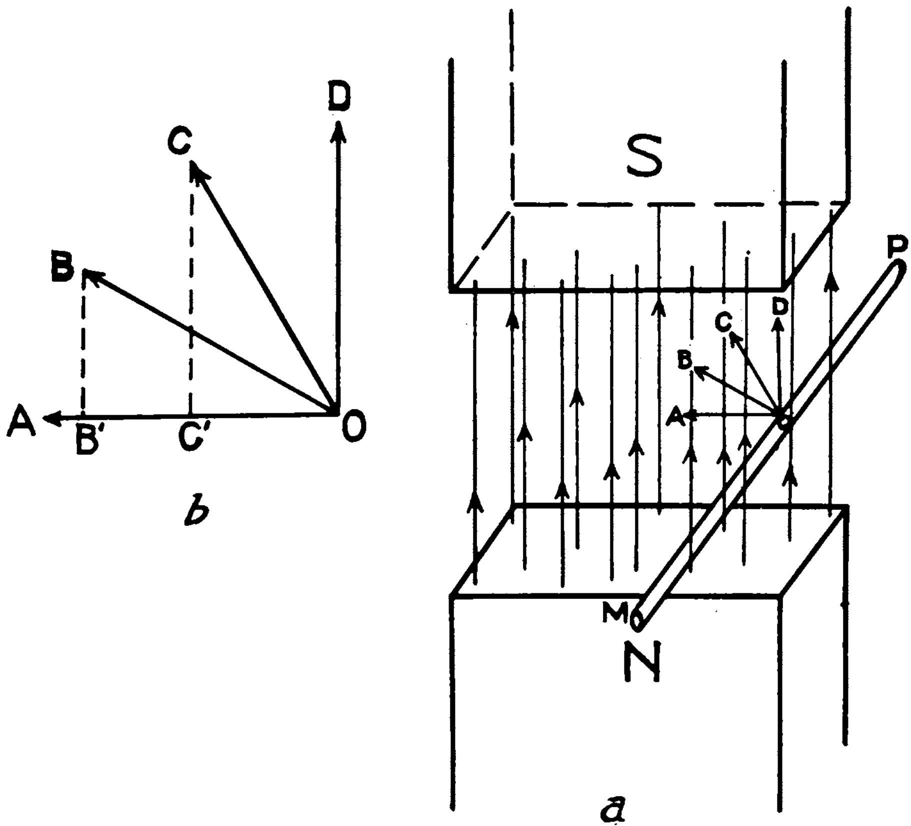

Fig. 3.

force, in the direction indicated by A, and that it has an e.m.f. of two volts induced in it. Now let MP be moved at the same speed in the direction of B. It is quite evident the conductor will not cut as many lines per second as when moving in the direction of A. If OA, Fig. 3b, represents the e.m.f. induced (two volts) when MP is moved in the direction of A, OB′ (about 1.77 volts) represents the e.m.f. induced when MP is moved in the direction of B. Reasoning similarly, the e.m.f. induced, when MP is moved in the direction of C, is represented by OC′ (about 1.155 volts); and when the conductor is moved in the direction of D, it is moving in the same direction as the lines of force and therefore does not cut any of them. Therefore, MP, when moving in the direction of D, has no e.m.f. induced in it. It is thus seen that the greatest e.m.f. is induced when a conductor cuts a magnetic field at right angles to the direction of the field; and that the induced e.m.f. continues to grow smaller as the angle between the direction of the field and the direction of motion becomes less, reaching the limit, zero, when the conductor moves in the same direction as the field. Exactly the same results as explained previously occur when the conductor is moved at angles below the horizontal. Accordingly, if MP moves in the direction of D or in a direction opposite to D, no e.m.f. will be induced, because, as stated before, no magnetic lines are cut.

To sum up the foregoing: The magnitude or strength of an induced e.m.f. is proportional to the rate of cutting lines of force. The rate of cutting flux depends on four factors: first, the strength of the magnetic field; second, the length of the inductor in the field; third, the speed of the inductor; and fourth, the angle at which the inductor cuts the magnetic field.

Example 4.

A wire eight centimeters long moves at right angles to a magnetic field at a speed of 1000 feet per minute, and has 2.25 volts induced in it. Suppose this wire had been twelve centimeters long and had moved at such an angle that, even though it moved as fast as at first, it would have cut but one half as many lines per second per unit length; and further, suppose the wire had moved in a field one fourth as strong as at first, and at a speed of 5000 feet per minute. What would have been the induced voltage in the second case?

Solution:

Because of a length of twelve centimeters, the conductor would have had $\frac{12}{8} \times 2.25$ volts induced in it.

Because of a movement not at right angles, it would have had $\frac{1}{2}$ of 2.25 volts induced.

Because of a field $\frac{1}{4}$ as strong, it would have had $\frac{1}{4} \times 2.25$ volts induced.

Because of a speed of 5000 feet per minute, it would have had $\frac{5000}{1000} \times 2.25$ volts induced.

These combined causes would have resulted in an e.m.f. of $\left(\frac{12}{8}\times\frac{1}{2}\times\frac{1}{4}\times\frac{5000}{1000}\right)\times 2.25$ volts $=\frac{15}{16}\times 2.25$ volts $= 2.109$ volts being induced in the inductor of the second case.

4. DIRECTION OF INDUCED E.M.F.—It has been mentioned that the direction of an induced e.m.f. (that is, the direction in which current would flow if the circuit were completed) is influenced by the direction in which the inductor cuts the magnetic field. Actually, the direction of an induced e.m.f. depends on two factors: first, the direction of motion of the inductor; and second, the direction of the field of the magnet. Just as the left-hand motor rule was used to indicate direction of motion, the *right-hand dynamo* rule will be used to indicate direction of e.m.f. When the thumb, first, and second fingers of the *right* hand are all extended at right angles to each other—the thumb indicating the direction in which the inductor moves, the first finger indicating the direction of the field—then the second finger will indicate the direction of the induced e.m.f. This rule may be more easily understood by referring to Fig. 4.

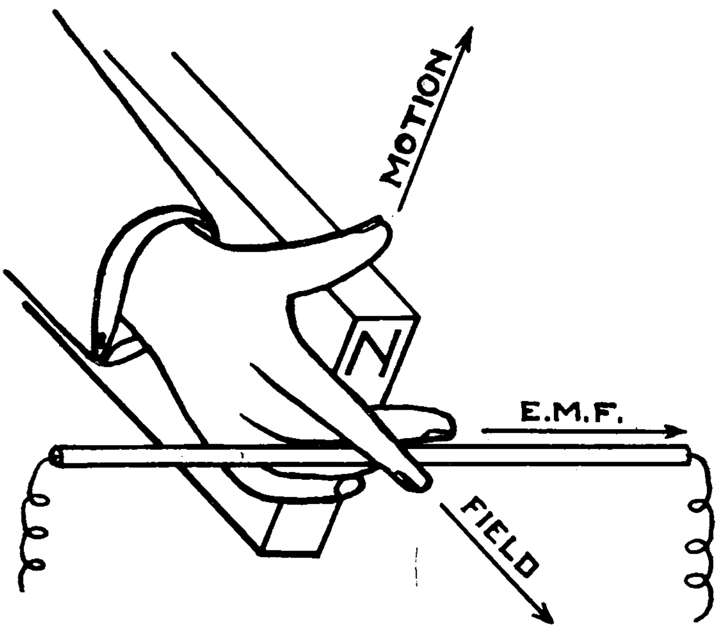

Fig. 4.

If the inductor is moved across the pole face of the bar magnet as illustrated, in the direction the thumb or arrow marked *motion* is pointing, and the flux is in the direction of the first finger or arrow marked *field*, an e.m.f. will be induced in the wire which will tend to force current in the direction indicated by the second finger or arrow marked *e.m.f.* Changing either the direction of the field or the direction of motion, changes the direction of e.m.f. If the inductor is stationary while the field moves, that is, if the wire of Fig. 4 is held still while the face of the magnet is moved in front of it, then, in order to determine the direction of the induced e.m.f., the thumb must point in the opposite direction to the movement of the magnet; that is, it points in the direction of the movement of the conductor with respect to the moving field.

Example 5.

Suppose the wire of Fig. 28, Pamphlet E-4, had no current flowing in it and were moved in the direction of arrow B. In what direction is the induced e.m.f.?

Solution:

Use the *right-hand dynamo* rule. Point the thumb in the direction of the movement of the inductor or in the direction of B; point the first finger in the direction of the field or toward the right of the figure; the second finger will then point toward the page, indicating that the induced e.m.f. is directed away from the reader. If the circuit were complete, current would flow into the page (not as shown in the figure).

Example 6.

In what direction is the e.m.f. in Fig. 2a?

Solution:

Point the thumb of the right hand in the direction of movement of the inductor, toward the left-hand side of the page; point the first finger in the direction of the field, toward the top of the page; the second finger, which indicates the direction of e.m.f., will then point into the page.

5. E.M.F. OF A LOOP MOVED ALONG A MAGNETIC FIELD.—In Fig. 5, a four-sided loop of copper wire is moved between two magnetic poles in the direction indicated. It will be readily seen that all four sides of this loop move parallel to the lines of force. The movement of AB and CD is exactly similar to the movement of MP, in Fig. 3a, when it moved toward D. Just as MP in Fig. 3a had no e.m.f. induced in it when moving toward D, AB and CD will have no e.m.f.'s induced in them when moving in the direction indicated, since no lines of force are cut.

Likewise, BC and AD have no e.m.f.'s induced in them, because they are moving parallel to, and hence do not cut, the magnetic flux. No matter how fast AB and CD are moved, there can be no e.m.f. induced in any part of the circuit.

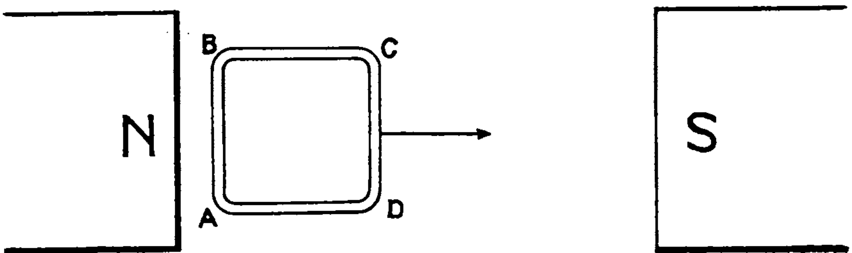

Fig. 5.

6. E.M.F. OF A LOOP MOVED ACROSS A UNIFORM MAGNETIC FIELD.—In Fig. 6, the loop of Fig. 5 is moved between two poles in the

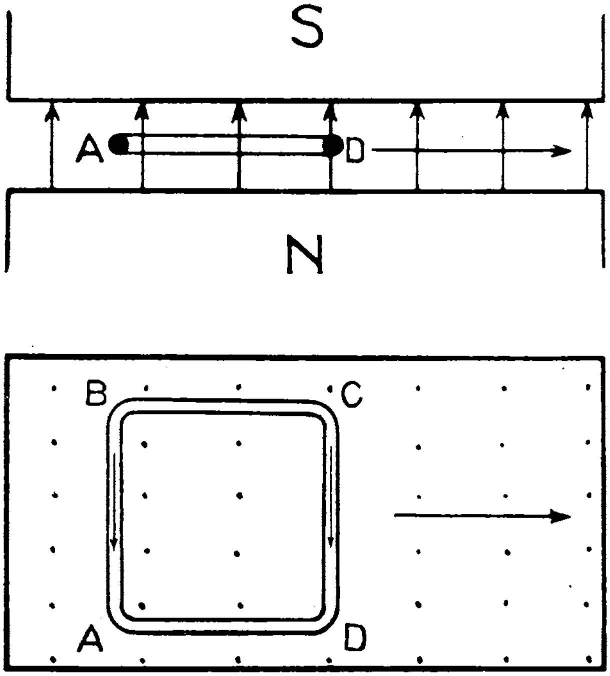

Fig. 6.

manner indicated. The magnetic field is assumed to be of the same intensity throughout. Here, it will be noticed that AB and DC cut lines of force and are therefore inductors, while BC and AD are parallel to the direction of motion and do not cut any magnetic lines. AB and DC will have equal e.m.f.'s induced in them in the directions shown by the arrows. The e.m.f.'s of AB and DC tend to force current in opposite directions through the closed loop; hence no current will flow. A similar case is where two primary cells of equal voltage are connected as illustrated in Fig. 7.

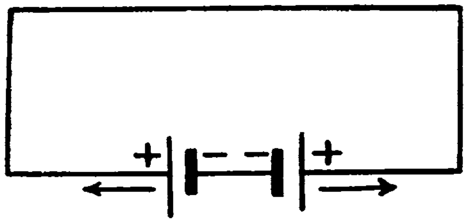

Fig. 7.

The two cells, attempting to send current in opposite directions, will neutralize each other and no current will flow. It must be borne in mind that even though no current is flowing in the loop of Fig. 6 or in the circuit of Fig. 7, the e.m.f.'s are still there just the same as though current were flowing.

A simple dynamo might be made from the loop of Fig. 6 by attaching an external circuit as shown in Fig. 8.

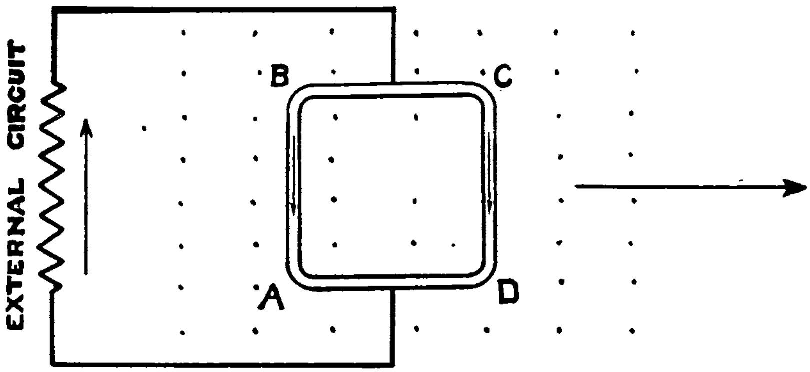

Fig. 8.

The two inductors, AB and DC, now act as two primary cells in parallel and force current through the external circuit in the direction indicated. Such a dynamo, however, is not practical because the loop, ABCD, would soon move beyond the limits of the largest pole face and no continuous e.m.f. could be generated.

7. E.M.F. OF A LOOP MOVED ACROSS A NON-UNIFORM MAGNETIC FIELD.—Suppose, in moving from the left to the right of the field in Fig. 6, the field encountered is of increasing strength; DC will then be continually cutting a stronger field than AB. Since the magnitude of induced e.m.f. depends on the strength of the field, DC will have a greater induced e.m.f. than AB, and as a result current will flow around the loop in a clockwise direction. For example, suppose DC has 1.25 volts induced and AB has 1.0 volt induced. The e.m.f. of DC, after overcoming the e.m.f. of AB, has 1.25 volts minus 1.0 volt, or 0.25 volt, remaining, which pressure forces current around the loop. If the resistance of the loop is one tenth of an ohm, the current flowing is, according to Ohm's law, $\frac{.25}{.10}$ amperes, or 2.5 amperes. Although but 0.25 volt is active in sending current through the loop, the total induced voltages are nevertheless existing in the inductors.

8. E.M.F. OF A LOOP ROTATED IN A UNIFORM MAGNETIC FIELD.—The e.m.f. of a loop rotating about an axis passing horizontally through its center will be considered as another illustration of electromagnetic induction by the movement of an inductor through a magnetic field.

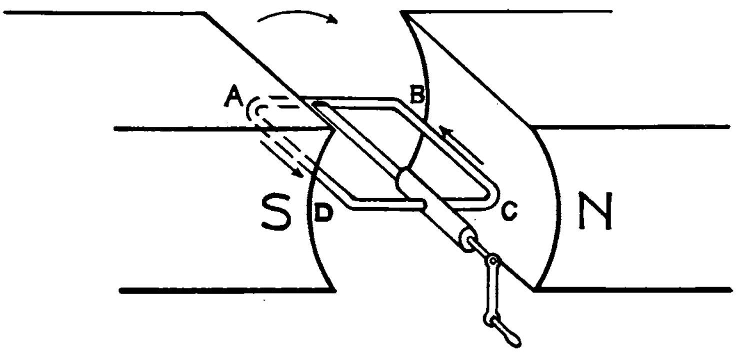

Fig. 9.

The coil of Fig. 9 is revolved in the direction indicated by the arrow at the top of the figure. By application of the right-hand dynamo rule, it can be seen that the e.m.f.'s induced in AD and BC are in the direction indicated by the arrows. Both e.m.f.'s tend to force current around the loop in a counter-clockwise direction; and these e.m.f.'s in AD and BC act exactly like two primary cells connected in series. It is important to note at this point that the loop does not always have an e.m.f. of the same magnitude induced, nor does a constant value of current flow, even though the loop is rotated at a uniform speed.

Fig. 10 is a cross-sectional view of Fig. 9 with the magnetic field between the poles, N and S, represented by straight lines equally spaced. It is seen that when the loop is in a vertical position, C′D′, both inductors are moving for an instant, parallel to the lines of force; that is, they are not cutting the lines. At this instant, therefore, there will be absolutely no e.m.f. induced, and no current will flow in the loop. As the loop moves away from this vertical position, it begins to cut across the magnetic field, cutting but few lines at first, but more and more as it moves toward the horizontal position, CD, at which position it cuts more lines in a given time than in any other position. Since

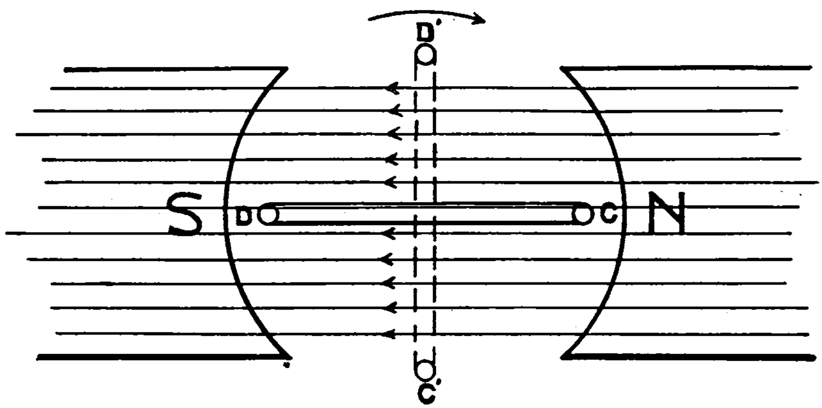

Fig. 10.

induced e.m.f. is proportional to the rate of cutting lines of force, the induced e.m.f. will be zero in the vertical position and maximum in the horizontal position. As the coil passes the horizontal, the e.m.f. begins to decrease and finally becomes zero when the vertical position is again reached; that is, when D′ in Fig. 10 has travelled to the bottom of its revolution and C′ to the top. It is assumed in this discussion that the field between the magnets is of the same strength throughout. Many different phenomena arise when the fields across the pole faces are of varying strengths.

It will be seen that the direction of current in BC (Fig. 9) is into the page, or away from the reader. The position of the loop and the direction of current in AD and BC after one half revolution of the coil has been completed are shown in Fig. 11.

The two sides of the coil are now cutting the lines of force in an opposite direction to that shown in Fig. 9; because in Fig. 9, AD was moving *up* and BC was moving *down* through the field, while in Fig. 11, AD is moving *down* and BC is moving *up* through the magnetic field. The direction of current in BC is therefore in the opposite direction to that shown in Fig. 9, or out of the page. Reasoning similarly, it is seen that the direction of current in AD is into the page.

The variation in the value of induced e.m.f. in the conductor AD can well be shown by a curve. Suppose AD (Fig. 9) were midway between the poles as D′ in Fig. 10. Here the voltage is zero. When AD moves to a position say 30° farther on, assuming a certain flux

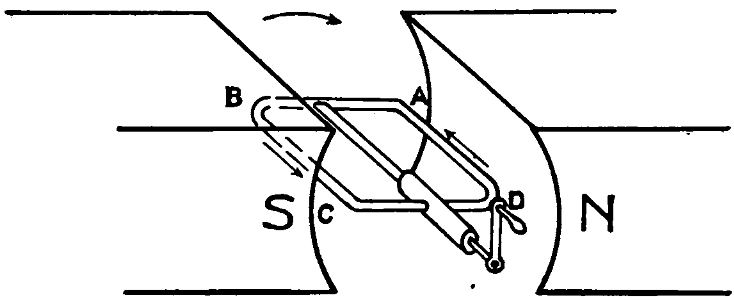

Fig. 11.

density and a certain speed for the inductor, the induced voltage is one volt; at 60° the voltage is 1.73 volts. Tabulating the different values of induced voltage, we have:

At	0°	the	induced	voltage	is	0.00	volts.
"	30°	"	"	"	"	1.00	"
"	60°	"	"	"	"	1.73	"
"	90°	"	"	"	"	2.00	"
"	120°	"	"	"	"	1.73	"
"	150°	"	"	"	"	1.00	"
"	180°	"	"	"	"	0.00	"

NOTE.—There are 360° in a circle or circumference.

Using degrees as abscissæ and volts as ordinates, the curve shown in Fig. 12 can be plotted.

The voltage of AD has now been traced through 180°, or until AD has moved from the top to the bottom of its path. Here the induced voltage is again zero. As AD continues and goes up the other, or left, side it is seen, from the right-hand dynamo rule, that the direction of the e.m.f. is changed; that is, it is now directed toward the reader, or

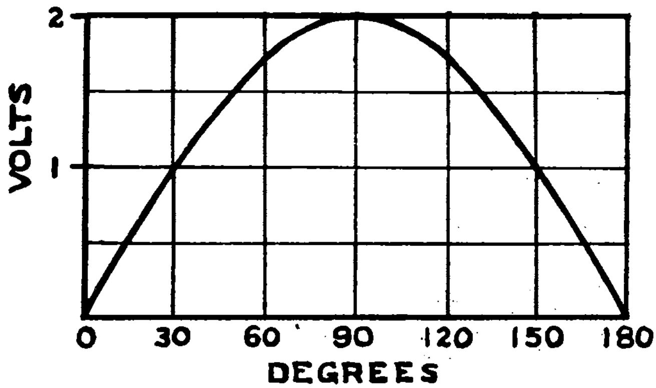

Fig. 12.

out of the page. The variation in voltage as AD moves upward is exactly the same as when it moved downward, except that the direction of the voltage and current has been changed. The voltage of AD moving down the right side was represented by positive ordinates above the base line in Fig. 12; the voltage of AD moving up the left side (having changed direction with respect to the magnetic field) can be represented by negative ordinates below the base line, as shown in Fig. 13.

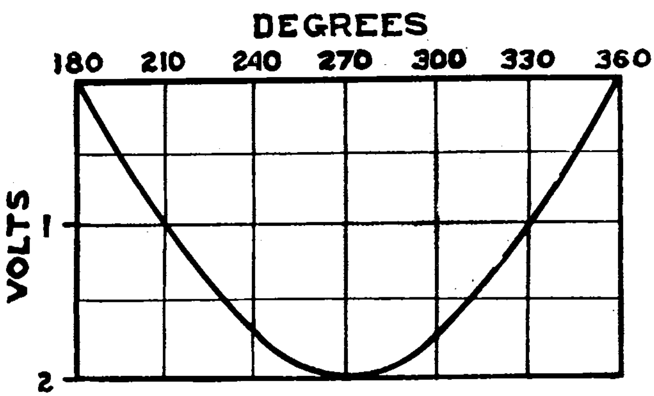

Fig. 13.

In this figure AD has moved from the bottom (the 180° point) to the top (the 360° or 0° point) where it started. At 360° or 0° the voltage is zero as at the start. The complete variation in voltage as AD makes a complete revolution can be represented by combining the curves of Figs. 12 and 13 as shown in Fig. 14.

The voltage of AD will start again and go through the same values as shown in Fig. 14 when the loop makes a second revolution. When the voltage has gone through a complete set of values as in Fig. 14, it is said to have made a *cycle*. Figs. 12 and 13 represent *half cycles* or *alternations*. In a complete cycle the voltage starts at zero, rises to a maximum positive value, falls to zero, reverses, rises to a maximum negative value and falls to zero again. By plotting Fig. 14 on cross-section, or co-ordinate, paper and selecting proper scales, the voltage at any point in the cycle can be easily ascertained from the curve.

The inductor BC of Fig. 9 goes through the same cycles as AD. Its

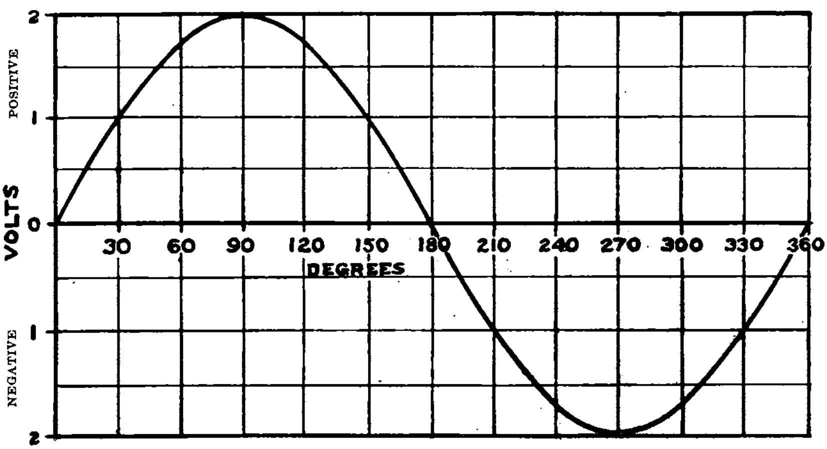

Fig. 14.

voltage is always in such a direction that it adds to the voltage of AD. When the voltage in one portion of the loop is clockwise, the voltage in the other is also clockwise, and vice versa. It is readily seen that CD and AB have no e.m.f.'s induced in them, because they move in a plane parallel to the lines of force, and therefore do not cut any flux, which cutting is necessary to induce an e.m.f.

When the coil in Fig. 9 is rotated rapidly, the voltage induced in the coil goes very rapidly through the cycle as shown in Fig. 14. The current flowing in the coil, because of the changes in the induced e.m.f., changes direction twice in every cycle. Thus the current alternates in direction, and therefore an *alternating current* is produced in the coil. A voltmeter placed across the coil would not show the maximum or the minimum voltage, but rather an intermediate voltage. In Fig. 12 the voltmeter would read about 1.41 volts.

Very frequently the abscissæ of curves, similar to the one illustrated in Fig. 14, represent time in hundredths or thousandths of a second. If

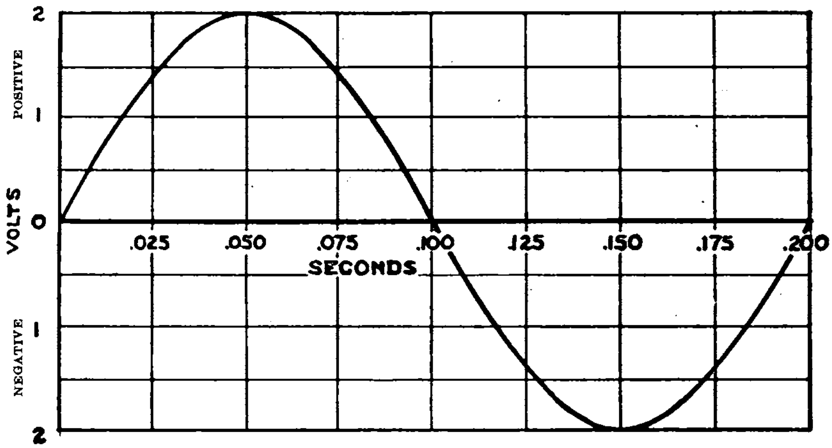

Fig. 15.

the coil in Fig. 9 were rotated at a speed of 300 rev. per min. (approved abbreviation for revolutions per minute, but sometimes written r.p.m.), the e.m.f. would make a cycle in $\frac{1}{300}$ of a minute or $\frac{1}{5}$ of a second. The total length of the horizontal values shown as 360° in Fig. 14 would then represent $\frac{1}{5}$ of a second. The curve plotted with time as abscissae is as shown in Fig. 15.

It is seen that there would be 5 such cycles made in a second; or the *frequency* is 5 cycles per second. The frequency of many alternating current lighting and power circuits is 60 cycles per second; 25 cycles per second is also a common frequency for power systems. The symbol ~ is sometimes used for cycle, and frequencies of 25 cycles or 60 cycles would be written 25 ~ or 60 ~. Alternating currents with frequencies as high as 10^9 or 1,000,000,000 cycles per second, are used in wireless telegraphy.

9. E.M.F. AT ENDS OF INDUCTOR WHEN CURRENT FLOWS.—In example 1, Section 3, an e.m.f. of 3.6 volts was induced. Suppose an external circuit of 5 ohms is connected to the inductor, and assume the resistance of the inductor is 1 ohm. The total resistance of the circuit is then 6 ohms, and the current flowing is $\frac{3.6}{6}=0.6$ ampere. While this current is flowing, a voltmeter placed across the ends of the inductor will not read 3.6 volts as it would if no current were flowing. It must be remembered that, since the rate of cutting lines is still the same, the induced e.m.f. is still the same, 3.6 volts. However, just as in Pamphlet E-2, it was brought out that the voltage generated by a dynamo is considerably reduced when it reaches the distant customer, because of an IR drop in the line wires; so, here, the voltage at the terminals of the inductor is less than the induced e.m.f. In this case the IR drop is $0.6 \times 1 = 0.6$ volt, and the voltage across the terminals, and hence the voltage which acts on the external circuit, is 3.6 volts minus 0.6 volt, or 3 volts. So long as no current flows, the induced voltage is the voltage existing at the terminals; but the instant current flows, the terminal voltage falls below the induced voltage. Even a voltmeter cannot measure exactly the induced voltage, because this instrument takes a small amount of current and measures only the terminal voltage. A voltmeter, however, because of the extremely small current it uses, indicates the induced voltage close enough for all practical purposes. It is thus seen, in part, why the terminal voltage of a generator decreases as the load, or current, increases, even though the induced voltage remains the same. This discussion can be stated by the following formula:

$$E_t = E_i - IR_i \qquad \text{Eq. (1).}$$

in which E_t is the terminal voltage of any conductor; E_i, the induced voltage of the inductor; I, the current flowing; R_i, the resistance of the inductor.

Example 7.

If the induced voltage of a winding or an inductor is 15 volts; the current flowing, 5 amperes; the resistance of the winding, 2 ohms; what is the terminal voltage?

Solution:

By substituting the known values, $E_i = 15$; $I = 5$; $R_i = 2$, in Eq., (1), the terminal voltage, E_t, is found as follows:

$$E_t = 15 - (5 \times 2)$$
$$= 15 - 10$$
$$= 5, \text{ the terminal voltage.}$$

10. LENZ'S LAW OR MAGNETIC PULL.—It was stated in Section 2 that it required no more energy to move AB (in Fig. 2) than to move so much rope in air. Now join A and B externally with a conductor and cause current to flow. One of the results which appears has just been discussed; namely, the terminal voltage of AB is reduced. Another important change takes place: the wire AB is no longer like so much rope in air; it requires an appreciable effort to force AB through the magnetic field. Indeed, with a little thought, this result could have been predicted. When AB (Fig. 2a) is moved toward the left, an e.m.f. is induced, which, according to the right-hand dynamo rule, would cause current to flow into the page. Here then is a wire with current flowing into the page, in a field moving toward the top of the page. The wire will assuredly have a side push on it just as the wire of Fig. 28, Pamphlet E-4, had a side push on it. Use the left-hand motor rule to find the direction of this force. Pointing the first finger toward the top of the page, the second finger into the page, it is seen by the direction in which the thumb points that the wire is pushed toward the right. In reality, the operator is moving the wire to the left; but while he is doing so there is a contrary force tending to force the wire to the right. This contrary force is often spoken of as the *magnetic pull* or the *motor action* of an inductor. It therefore takes much more power in an engine to drive the drum of a generator around in a magnetic field than it takes to drive a simple wheel of the same weight not in a magnetic field. In the latter case the engine merely works against the friction of the bearings, while in the former case the engine works against the friction of the bearings and also the much greater force, the magnetic pull or the motor action, which tends to rotate the drum in a direction just opposite to the direction it is moving.

The above results could have been expected looking at them from another viewpoint. If an inductor or a generator did not have to work against the "contrary force," all its power would be used in overcoming the friction of the parts in the air and in the bearings. But power to run motors and thence machinery is obtained from generators. From where would this power come? It could not be the power expended in overcoming friction, and, if this were all the power the engine were giving, it would be puzzling to understand from where the power given out by a generator came. The conclusion must be reached that the engine was expending power in driving the inductors on the drum through the magnetic field, and this mechanical power exerted by the engine appears as electrical power at the terminals of the generator. Mechanical energy can thus be transformed into electrical, or vice versa; but in all cases more energy must be put into the transforming apparatus—whether it be a generator, motor or transformer—than is taken out, on account of the losses occurring in the transforming apparatus.

The above phenomenon was very early studied by Lenz. He merely applied the general law of CONSERVATION OF ENERGY (energy cannot be created or destroyed) to electrical phenomena, and gave the following law, which is called *Lenz's law: When electric currents are induced by the motion of a conductor through a magnetic field, the induced currents set up a field in such a direction that their magnetic effect tends to stop the motion, or to oppose the lines producing the motion.*

OTHER METHODS OF INDUCING E.M.F.'S

11. A BAR MAGNET THRUST INTO A COIL.—That which amounts to the same thing as moving a wire in front of a magnetic pole is moving a

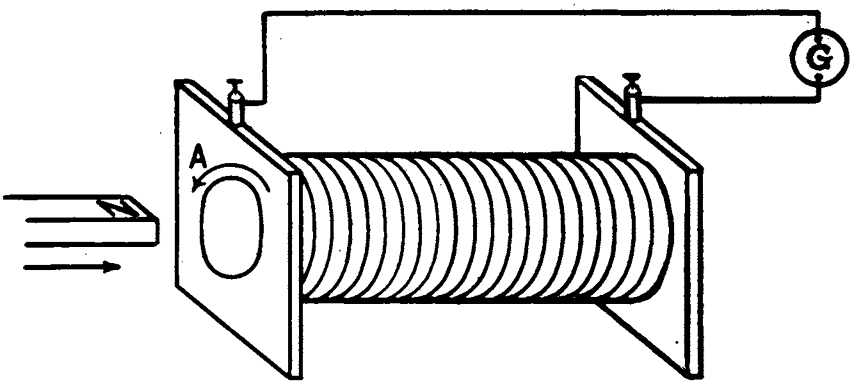

Fig. 16.

magnetic pole near a wire. If, in Fig. 16, the bar magnet is thrust into one end of the coil, the flux issuing from the N pole will cut the turns of the coil and will cause an e.m.f. to be induced in the coil. This is easily made evident by attaching the ends of the winding of the coil to a galvanometer. By application of the right-hand dynamo rule (the thumb will point to the left of the page), it is seen that current flows around the coil as indicated by arrow A; that is, when the circuit is closed through the galvanometer, G. Applying the rule for determining the polarity of a coil, as stated on page 10, Pamphlet E-4, it is seen that

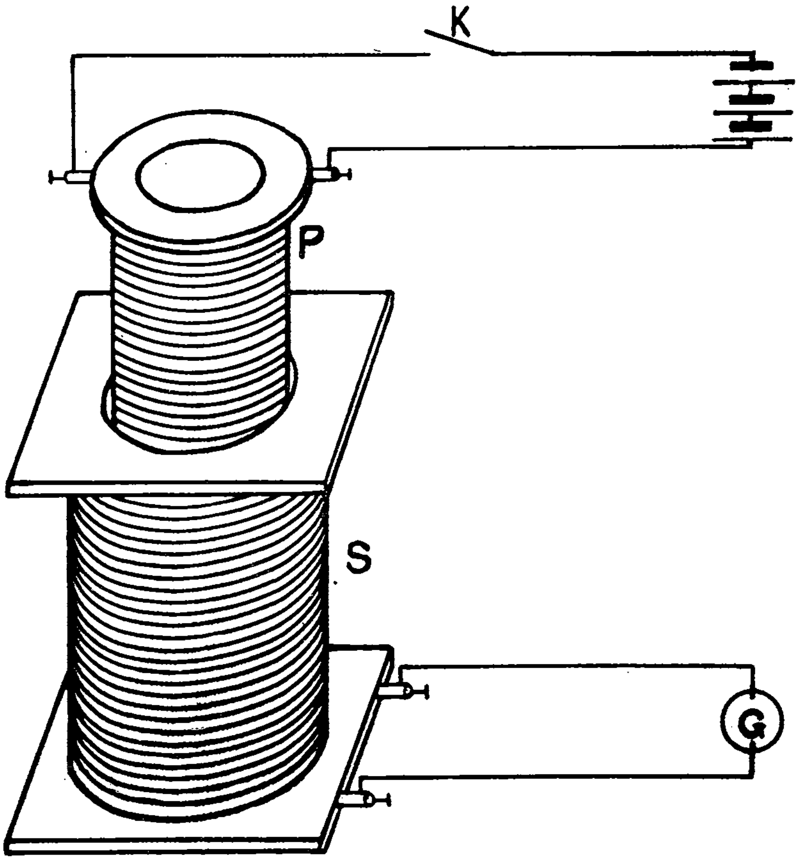

Fig. 17.

the left end of the coil is an N pole. Therefore, thrusting a north pole into the end of a coil induces a pole of similar polarity on that end. Since like poles repel, a force acts on the bar magnet tending to oppose its introduction into the coil. If the external circuit of the coil is open, no current will flow; hence, there will be no field to repel the magnet. The above is but another illustration of Lenz's law, that induced currents tend to oppose the motion producing them. Likewise, when the magnet is drawn out of the coil, an S pole is induced at the end; and a force is exerted to hold the magnet in the coil; that is, the induced currents again oppose the motion producing them.

Instead of using a bar magnet to thrust into a coil to induce current, another coil may be used. If, upon closing the key, K, current in coil P, Fig. 17, viewed from the top, is flowing in a clockwise direction,

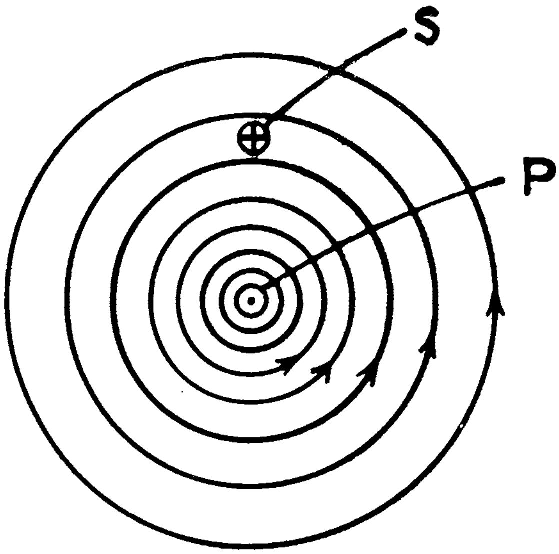

Fig. 18.

the end in the coil S, is a north pole. While thrusting this end into the coil S, a north pole is induced at the upper end of S. The coil P is called the *primary* coil and is the prime source of energy; the coil S is called the *secondary* coil, and depends on P for its energy. It must be clearly kept in mind that as soon as the motion of coil P stops (assuming the current in P is constant), there will be no e.m.f., and hence no current, induced in S. Again, if P is kept still and S moved over it, the same action will take place as when P is thrust into S.

12. MUTUAL INDUCTION.—Let the circuit of P be opened at K and let coil P, in Fig. 17, be thrust completely within coil S. Consider what happens when K is suddenly closed. The circular field around each turn of wire in P begins to spread out and tends to produce a field similar to the one shown in Fig. 1, Pamphlet E-4. In Fig. 18 this field is shown surrounding the conductor, P, which represents a cross-sectional view of one side of a turn of wire in the primary coil of Fig. 17, while S represents a similar view of one side of a turn of wire in the secondary coil of Fig. 17. The circular loops start from the very center of the wire in infinitely small circles and spread out at a definite speed. The circular field of P, in sending out its circular loops, cuts the wires of S which are directly opposite it. Accordingly, the wires of S will have an e.m.f. induced in them. Since the e.m.f. of P is directed out of the page, either by Lenz's law or by the right-hand dynamo rule, it is seen that the e.m.f. of S is into the page. Thus, if current in P (Fig. 17) is suddenly started to flow in a clockwise direction, an e.m.f. will be set up in S which will force current in a counter-clockwise direction. As the current in S grows, its field grows and opposes the field of P. The induced current in S, therefore, retards the rise of current in P. As the current in P approaches a steady value, the field of P becomes stationary; and since the e.m.f. in S depends on the changing field of P, the induced e.m.f. in S soon dies down. If, now, K is opened, the field of P will contract and again cut the wires of S, but in the opposite direction. Now the induced e.m.f. of S is clockwise and its current produces a field which is in the same direction as the field of P. The current in P tends to die down while the induced current in S tends to keep it up—thus again opposing the action of P. If K is opened and closed intermittently, an alternating current will be induced in S. The e.m.f. across the terminals of S depends on the number of turns of wire in S—the greater the number of turns the greater the induced e.m.f. Very high voltages can be produced in a secondary coil by making the number of turns on it very great. It need hardly be said that an alternating e.m.f. impressed on P will have an effect on S very similar to the interrupted direct current above described. In fact, the alternating current transformer is but an arrangement of a primary and a secondary coil to transform the applied voltage to higher or to lower voltages.

A very common use for induction coils is in the ignition in gas and gasoline engines. The spark gap, A, in Fig 19, is in the engine cylinder where it is desired to explode the gas at certain instants. At the proper time, the primary circuit is broken at contact B, which contact rotates proportionally with the shaft. Breaking the primary circuit induces a very high e.m.f. in the secondary circuit and causes a hot spark to jump across the gap A. This spark ignites the gaseous mixture in the cylinder, thus causing the mixture to explode.

13. SELF-INDUCTION.—A single coil such as shown in Fig. 6, Pamphlet E-4, has induced e.m.f.'s in it when an e.m.f. is suddenly impressed across its terminals. A large current does not flow in a coil the instant the circuit is closed. It takes an appreciable time for a current to reach its full value in a large coil with an iron core. This phenomenon is known as *self-induction*, or *inductance*, the unit of which is called the *henry* (named after Joseph Henry). It corresponds to the cutting of 10^8 magnetic lines when one ampere is turned on and off; that is, when

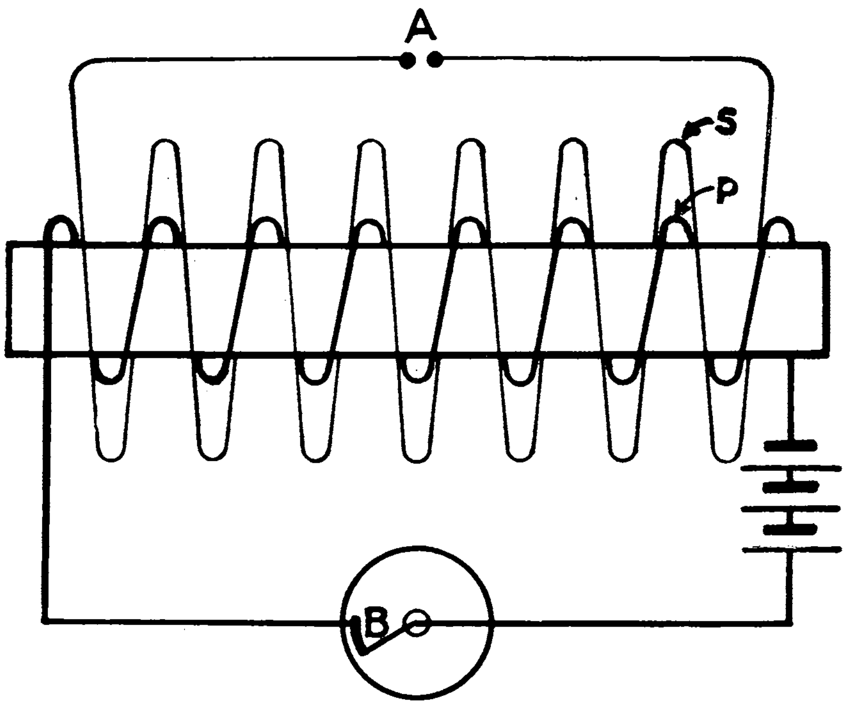

Fig. 19.

the e.m.f. induced in a circuit is one volt, while the inducing current varies at the rate of one ampere per second, the induction is one henry. In other words, a conductor has one henry induced in it when it is so constructed that a rate of change of current of one ampere per second in it requires the expenditure of one volt. The symbol for inductance is the letter L.

By referring to Fig. 20 this phenomenon can be further explained.

A current is made to enter at M. It first reaches the coil at the wire A_1. The growing field around A_1 cuts A_3, and, as seen by the right-hand dynamo rule, induces an e.m.f. in A_3, which tends to send current *out.* But the current entering at M goes *in* at A_3, and the growing current going *in* at A_3 is thus opposed by the *out* e.m.f. induced by the growing field around A_1. Similarly, the current in A_5 is opposed by an e.m.f. induced by the growing field around A_3; and the current in A_4 by the growing field of the current in A_2, etc. It is thus seen that a growing current in such a coil is "choked" back, and it requires some measurable length of time for the current to come up to the full value as would be found by Ohm's law.

In a like manner, when the circuit is broken and the field around the wires and within the coil starts to die out, the wires are again cut

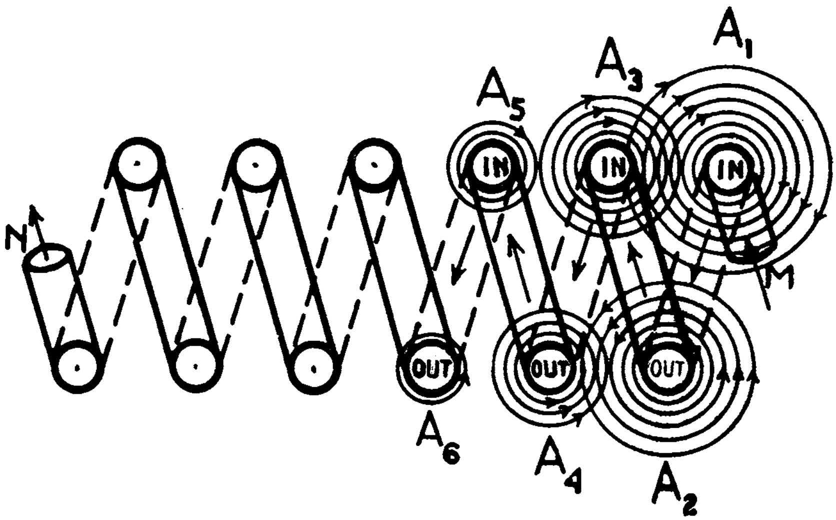

Fig. 20.

by the collapsing field, though this time in a direction which sets up an e.m.f. tending to maintain the field and the current as it is. If the magnetic field is sufficiently strong and the circuit is broken suddenly, the voltage induced by the collapsing field cutting the wires of the coil may be many times the original impressed voltage. This would account for the spark jumping across the breaking point when the current in such a coil is suddenly interrupted.

There is also magnetic induction even in a straight wire when current starts or stops suddenly in that wire. When current starts, the circular magnetic lines springing from the center of the wire cut the wire as they spread out, and similarly cut the conductor in the opposite direction when the circuit is broken and the field collapses. When the current starts, the induced e.m.f. is in a direction opposite to the impressed e.m.f. and tends to impede the growing current. When the circuit is broken, the induced e.m.f. created is in the same direction as the impressed e.m.f. and tends to keep the current flowing. As always, the induced e.m.f. tends to prevent any change from existing conditions; that is, when current starts to flow the induced e.m.f. tends to hold to the old condition of no current, and when the flow of current tends to stop because of a broken circuit, the induced e.m.f. tends to keep the current flowing. Of course, the self-induction in a straight wire is very insignificant and to all practical purposes the instant an e.m.f. is impressed on the wire the current has a value as found by Ohm's law. Theoretically, however, it takes some time for the current to reach its full value in any circuit, the time ranging from less than a millionth of a second in a small straight wire to several seconds in a large coil with an iron core.

PROBLEMS

NOTE.—These problems should be answered one or more complete lessons at a time.

FIRST LESSON

1.—State and explain Faraday's principle.

2.—What is the meaning of *induced?*

3.—What is an *inductor?*

4.—If the inductor in Fig. 1 were moved across the pole face away from the reader, would the current flow to the right or to the left? Explain in full how the answer was obtained.

5.—If AB in Fig. 2a moves toward the bottom of the page, will there be an e.m f. induced in it? Give a reason for the answer.

6.—Can there be an e.m.f. induced in a wire when it is not moving? Explain.

7.—Hold a wire in a horizontal position east and west and move it up quickly. Will there be an e.m.f. induced in it? If so, in what direction is it? Explain how the answer was obtained.

8.—Name four factors on which the magnitude of induced e.m.f. depends.

9.—How many lines of force must an inductor cut in one twentieth of a second to have an induced e.m.f. of (*a*) 1 volt; (*b*) 0.1 volt?

SECOND LESSON

10.—Suppose the inductor in problem 9 (*a*) moves at the same speed in a similar field for one tenth of a second; what is the induced voltage? Explain the answer.

11.—When an e.m.f. is induced in an inductor, does current flow? Explain.

12.—A wire, 50 cm. long, is passed across a certain pole face in one tenth of a second. The pole face is 30 cm. by 30 cm. and has a flux density of 10,000 lines per square centimeter. What voltage is induced?

13.—(*a*) When will the voltage in the wire in problem 12 reach its greatest value?

(*b*) How long will the voltage exist?

(*c*) When will it be zero again?

14.—A wire, in cutting across the field of a magnet having a flux density of 5000 lines, has 0.2 volt induced in it. What would be the flux density of this magnet if it had caused an induced e.m.f. of one volt? Assume the wire moves at the same speed as in the first case.

15.—A wire cuts across the face of a magnet, having a flux density of 4000 lines, and has 0.06 volt induced in it. What voltage would have been induced had the flux density been 12,000 lines per square centimeter?

16.—Suppose MP, in Fig. 3, had 0.2 volt induced in it when moving in the direction of A. What e.m.f. would be induced when the conductor moves at such an angle that—even though it moves as fast as at first—it cuts but one fourth as many lines per second?

17.—A voltage of 0.32 volt is induced in a wire which is 14 cm. long moving at a speed of 1500 feet per minute in a field with a flux density of 3000 lines per sq. cm. What voltage is induced in a wire 15 cm. long moving in a field of 5000 lines per square centimeter at a speed of 5000 feet per minute?

18.—A wire, 10 cm. long, moves in a field, having a flux density of 15,000 lines per sq. cm., at a speed of 5000 feet per minute. What voltage is induced in the wire?

Third Lesson

19.—Twenty conductors are connected so that their induced pressures are in series, and cut through 5,000,000 lines of force at the constant rate of 3000 times per minute. What is the pressure induced by the set of conductors?

20.—If the coil of Fig. 9 has 3,000,000 lines of force going through it when it is in a vertical position, what voltage is induced in the coil when it rotates at a speed of 1000 revolutions per minute?

21.—(*a*) If the coil of problem 20 had consisted of 30 turns in series, what would have been the pressure induced?

(*b*) If the total resistance of the coil were 6 ohms, what current would flow, assuming the ends were connected together?

22.—What horsepower must be expended to turn the coil of problem 21, assuming no friction?

23.—The N pole of a bar magnet is thrust through a steel key ring and quickly withdrawn; the action is then repeated several times and the ring becomes warm. How could this be explained?

24.—Facing the ring of problem 23 when the magnet is thrust in from the near side, what can be said of the direction of current in the ring; is it flowing clockwise or counter-clockwise? Give two methods for obtaining the answer.

25.—The two ends of a large reel of bell wire are connected together. It is found that it takes more muscular effort to quickly insert and remove an electromagnet from the center of the reel when the two ends are together than when the two ends are free. Explain this fully.

26.—An electromagnet is inserted in a coil of wire and the terminals of the latter are then joined to a galvanometer. The galvanometer shows that no current is flowing. Why is this?

27.—If the switch controlling the current of a heavy electromagnet is suddenly opened, a large flame will arc across the contacts. Explain this phenomenon.

Fourth Lesson.

28.—The S pole of a magnet is suddenly drawn out of a coil whose ends are connected together. What is the polarity of the end of the coil where the magnet is pulled out? How was the answer obtained?

29.—The induced e.m.f. in a coil similar to that shown in Fig. 9 passes through the following values in each cycle when rotated at a speed of 400 rev. per min.:

0°	induced	e.m.f.	0.00	volts.
45°	"	"	+2.12	"
90°	"	"	+3.00	"
135°	"	"	+2.12	"
180°	"	"	0.00	"
225°	"	"	−2.12	"
270°	"	"	−3.00	"
315°	"	"	−2.12	"
360°	"	"	0.00	"

Plot a cycle using:

(*a*) Volts as ordinates (values on the vertical scale) and degrees as abscissæ (values on the horizontal scale).

(*b*) Volts as ordinates and time as abscissæ.

30.—What is meant by an alternating current?

31.—What is the frequency of the current in problem 29?

32.—(*a*) Is the induced e.m.f. always the voltage at the ends of an inductor? Explain in full.

(*b*) What is terminal voltage and how may it be calculated?

33.—An inductor of 0.2 ohm resistance has an e.m.f. of 0.1 volt induced in it. What is the terminal voltage when 0.05 ampere is flowing?

34.—State Lenz's law as you understand it. Give an example to illustrate this law.

35.—(*a*) If the secondary coil of Fig. 18 is open, will there be an e.m.f. induced in it when P is thrust into S?

(*b*) Will there be an upward thrust on P as it is pushed downward?

36.—If current in a coil is flowing clockwise and the end of the coil viewed is thrust within a second coil whose circuit is closed:

(*a*) In what direction is the current flowing in the second coil?

(*b*) By what two methods could the answer be obtained?